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A Mathematical Model Analysis of Smoking Tobacco in the Case of Haremaya Town; Ethiopia

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ABSTRACT

In this work we consider a deterministic compartmental mathematical model. We adopt and extend the O.Sharomi and A.B. Gumel mathematical model [13] on the spreading and control of tobacco smoking. We study by adding smoking cause death rate and one more compartments exposed class. We derived the smoking generation number R_s of the extended model which depends on five parameters. Using real data collected from Haremaya town, we found that the smoking generation number $R_s = 1.04184235$. This shows that the smoking generation number is greater than unity which in principle implies that the smoking habit of the community in Haremaya town is very high. To control the smoking habit we then identify the control parameter which gives insight to give up smoking. The basic control parameters that can decrease the number of smokers are the rate of transmission from nonsmoker class into exposed class $\beta = 0.65$. Other control parameters are also investigated and how they affect the smoking generation number to be less than one also discussed in detail in their sub sections. Together with the smoking generation number we also investigate the smoking free equilibrium point and smoking present equilibrium point. Using Ruth – Hurwitz stability criterion we have discussed their stability analysis.

Keywords: *Dynamical system; Smoking generation number; Equilibrium points; Stability analysis; Numerical analysis*

INTRODUCTION

Lung cancer is the leading cause of cancer deaths. Cigarette smoking causes 85% of lung cancer deaths. Research and statistics show that the major agent of lung cancer is tobacco smoke [1]. Tobacco smoking is by far the leading cause of lung cancer and the risk increases with the number of cigarettes smoked and the number of years spent on smoking. The U.S. Surgeon General estimates that 90% of lung cancer deaths in men and 80% in women are caused by smoking. Nonsmokers have a 20% - 30% greater chance of developing lung cancer if they are exposed to second hand smoke at home or work and exposure to secondhand smoke causes approximately 3400 lung cancer deaths among nonsmokers each year [15, 16]. Cigarette smoke is responsible for a great proportion of deaths within tobacco smoke. Each year in the United States approximately 400000 people die from cigarette smoke which accounts for one in every five deaths in the United States [10, 15]. Smoking is also an important risk factor for communicable diseases such as tuberculosis and lower respiratory infections [5, 6]. In a study done [5,8] shows that for seventh grade students were studied, with their first smoking experience and they found out that the most common factors leading students to smoke is cigarette advertisements. Moreover, smoking by parents, siblings/brothers or sisters/ and friends also encourage students to smoke [14]. Based on the relationship between lung cancer and cigarette smoke, we want to show the reduction of contact between nonsmokers and smokers and how to decrease the rate at which nonsmokers and smokers progress towards lung cancer. The likelihood that a smoker will develop lung cancer from cigarette smoke depends on many aspects such as the age at which smoking began, how long the person has smoked, the number of cigarettes smoked per day and how deeply the smoker inhales [1,9,11]. Smoking cessation programs are an important strategy for preventing the adverse outcomes and related costs of smoking during pregnancy [12]. Mathematical modeling plays an important role in understanding the complexities of an infectious diseases and their control. It can be beneficial for studying the mechanism underlying observed epidemiological patterns assessing the effectiveness of control strategies and predicting epidemiological trends [3].The work [4] proposed a simple mathematical model for giving up smoking. The authors considered a system with a total constant population which is divided into four compartments. The first compartment is potential smokers, people who do not smoke yet but might become smokers in the future and they denote it by P, the second compartment is smokers denoted by S, and the third one is those smokers who have quit smoking temporarily and denote by Q and the fourth compartment is those smokers who have quit smoking permanently and denote it by R.

In this study we consider a nonlinear system of differential equations is used to model the dynamics of a population which includes smokers. A general epidemiological model is presented to describe the dynamics of drug use among adolescents specifically tobacco use. Specific models are derived by considering other factors that have been identified to have an effect on the growing trend of tobacco use. The factors considered are peer pressure, relapse, counseling and treatment. Compartmental epidemiological models are used by a wide community of epidemiologists, public health officials and scientists to model how diseases spread and what strategies of control are most likely to succeed in case an epidemic occurs within a given population. Compartmental models segregate a population into distinct groups namely nonsmokers $P(t)$, smoker's $S(t)$, smokers who temporarily quit smoking $Q(t)$ and smokers who permanently quit smoking $R(t)$.

The total population represented by N at time t is given by $N(t) = P(t) + S(t) + Q(t) + R(t)$. Where $P(t)$ is a class of nonsmokers is increased by the recruitment of individuals into the nonsmoking class at a rate bN. It is assumed that non smokers inter the smoking class through effective contacts with smokers at a rate $\beta = cq$ where c is the average number of contacts per unit time and q is the probability of becoming smoker for a member of the P class. $S(t)$is the population of smokers is increased when nonsmokers begin to smoke and when smokers who temporarily quit smoking revert to smoking at rate α. The population decreased when smokers quit smoking permanently at a rate $\gamma\sigma$ and natural death rate μ. $Q(t)$is a class of Smokers who temporarily quit smoking which is represented by a fraction of $1-\sigma$ of smokers who temporarily quit smoking at the rate γ. It is decreased by natural death rate μ and reversion to smoke at the rateof α. $R(t)$is a class of Smokers who permanently quit smoking. This compartment is represented by the remaining fraction σ of smokers who permanently quit smoking at the rate of γ and decreased by natural death at the rate of μ.

To obtain a realistic representation on the effects of smoking on a given population we adopted and extend a deterministic model done by [13] which is relevant to tobacco smoking in Haremaya town, Ethiopia. The model we decided to use in studying tobacco smoking is similar to the Susceptible – Exposed – Infectious – Recovered (SEIR) compartmental model [3]. In this deterministic model we looked at the dynamics of an ordinary differential equation model which is derived and analyzed both analytically and numerically using Win plot. In our dynamical system we obtained the equilibrium points and the smoking generation number R_s just like the basic reproduction number R_0 in epidemiology represents the rate that people get infected or addicted. In this work the role of smoking generation number $R_s$is to determine if smoking habit in the population will eradicate out or increase. Simulations of our model were conducted using parameters and estimations from real data. Through simulations the model is analyzed to obtain different situations that produce interesting results among the specific classes.

In this work we derived the smoking generation number $R_s$of the extended model which depends on five parameters. Using real data collected from our research site we found that the smoking generation number$R_s = 1.04184235$. This shows that the smoking generation number is greater than unity which in principle implies that the smoking habit of the community in Haremaya town is high. To control the smoking habit we then identify the control parameter which gives insight to give up or decrease smoking. The basic control parameters that can decrease the number of smokers are the rate of transmission from nonsmoker class into exposed class$\beta = 0.6234$. Other control parameters are also investigated and how they affect the smoking generation number to be less than unity also discussed in detail in their sub sections. Together with the smoking generation number we also investigate the smoking free equilibrium point and smoking present equilibrium point and we have discussed their stability analysis using Routh –Hurwitz stability criterion.

THE MATHEMATICAL MODEL

Our initial model [13] is represented by four ordinary differential equations. Our extended model is represented by five ordinary differential equations by adding one more compartment $\boldsymbol{E(t)}$ the exposed group of individuals in between the compartments of nonsmoker and smoker and the smoking cause death rate $\boldsymbol{\xi}$.

The probability rate of conversion from smoker class into temporarily quit smoking class is $\boldsymbol{\sigma}$ and the probability rate of conversion from smoker to permanently quit smoking class is $\mathbf{1-\boldsymbol{\sigma}}$. We assumed here that the natural birth rate $\boldsymbol{\mu_0}$is different from natural death rate $\boldsymbol{\mu}$.

Table1. *State Variables, Parameters and their descriptions*

State Variables	Parameters	Descriptions
$P(t)\ /\ N(t)$		Nonsmoker individuals at time t / fraction of $N(t)$
$S(t)\ /\ N(t)$		Smoker individuals at time t / fraction of $N(t)$
$Q(t)\ /\ N(t)$		Temporarily quit smoker individuals at time t / fraction of $N(t)$
$R(t)\ /\ N(t)$		Permanently quite smoker individuals at time t / fraction of $N(t)$
$N(t)/N(t)$		The total population which is constant at time t
	k	Rate of conversion from temporarily quit smoking to permanent quit smokers
	α_1	The rate of conversion from exposed to smoker class
	ξ	Smoking cause death rate
	μ_0	Natural birth rate of the population
	β	Rate of conversion from nonsmoker to smoker
	α	Rate of return back from temporarily quit smoking to smoker
	γ	Rate of removal from smoker to temporarily quit and permanent quit smokers
	σ	The probability of conversion from smoker to temporarily quit
	$1-\sigma$	The probability of conversion from smoker to permanent quit smokers
	μ	Natural death rate
	ξ	Smoking cause death rate
	R_s	Smoking generation number

The Flow Chart and the Corresponding Dynamical System of the Initial Model

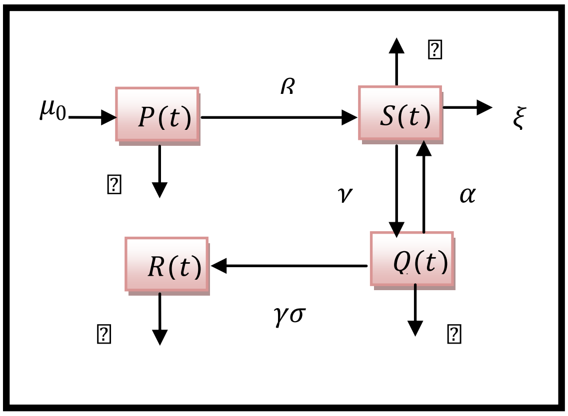

Figure 1. *Flow chart of the initial model constructed by [13]*

The corresponding dynamical system of the flow chart given by Figure 1is

$$\frac{dP(t)}{dt} = \mu_0 N(t) - \frac{\beta P(t)S(t)}{N(t)} - \mu P(t) \qquad (1)$$

$$\frac{dS(t)}{dt} = \frac{\beta P(t)S(t)}{N(t)} + \alpha Q(t) - \mu S(t) - \gamma S(t) - \xi S(t) \ (2)$$

$$\frac{dQ(t)}{dt} = \gamma(1-\sigma)S(t) - \mu Q(t) - \alpha Q(t) \ (3)$$

$$\frac{dR(t)}{dt} = \gamma\sigma S(t) - \mu R(t) \ (4)$$

After rescaling the dynamical system, the authors [13] found that:

The smoking free equilibrium point is

$(p, s, q, r) = (1,0,0,0)$,

The smoking present equilibrium point is

$$\big(p(t), s(t), q(t), r(t)\big) = \Big(\frac{b}{\mu(R_s - 1) + \mu}, \frac{\mu}{\beta}(R_s - 1), \frac{\mu\gamma(1-\sigma)}{\beta(\mu+\alpha)}(R_s - 1), \frac{\gamma\sigma}{\beta}(R_s - 1)\Big)$$

The smoking generation number is

$$R_s = \frac{\beta(\mu+\alpha)}{\mu(\mu+\gamma+\xi) + \alpha(\mu+\xi+\gamma\sigma)}$$

The Extended Model

Based on the assumptions we have mentioned above we do have a flow chart of the extended model below

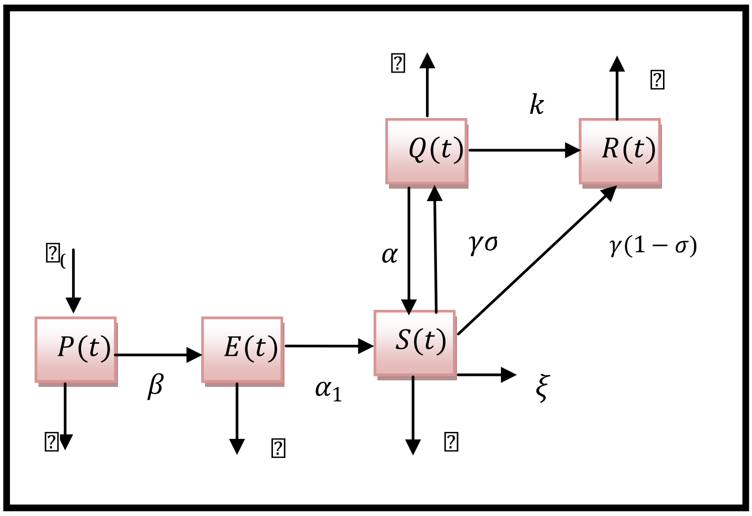

Figure 2. *The extended model flow chart*

The corresponding dynamical system of the extended model is

$$\frac{dp(t)}{dt} = \mu_0 N(t) - \frac{\beta P(t)E(t)}{N(t)} - \mu P(t) \quad (5)$$

$$\frac{dE(t)}{dt} = \frac{\beta P(t)S(t)}{N(t)} - \mu E(t) - \alpha_1 E(t) \quad (6)$$

$$\frac{dS(t)}{dt} = \alpha_1 E(t) + \alpha Q(t) - \mu S(t) - \gamma S(t) - \xi S(t) \quad (7)$$

$$\frac{dQ(t)}{dt} = -\alpha Q(t) + \gamma\sigma S(t) - kQ(t) - \mu Q(t \quad (8)$$

$$\frac{dR(t)}{dt} = kQ(t) + \gamma(1-\sigma)S(t) - \mu R(t) \quad (9)$$

Determination of Smoking Generation Number of the extended Model

The two important concepts in modeling outbreaks of infectious diseases are the basic reproductive number, universally denoted byR_0, and the generation time (the average time from symptom onset in a primary case to symptom onset in a secondary case), which jointly determine the likelihood and speed of epidemic outbreaks [3, 7, 17].In the dynamical system (5)-(9) the rate of appearance of new infections $\mathcal{F}$ and the transfer rate of individuals $\mathcal{V}$ at the disease free steady state $(N, E, S, Q, R) = (N(t), 0, 0, 0, 0)$ where $N(t) = P(t) + E(t) + S(t) + Q(t) + R(t)$ is

$$\mathcal{F} = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix}, \mathcal{V} = \begin{pmatrix} \mu+\alpha_1 & 0 \\ -\alpha_1 & \xi+\mu+\gamma \end{pmatrix} and thus \mathcal{V}^{-1} = \begin{pmatrix} \frac{1}{\mu+\alpha_1} & 0 \\ \frac{\alpha_1}{(\xi+\mu+\gamma)(\mu+\alpha_1)} & \frac{1}{\xi+\mu+\gamma} \end{pmatrix}$$

The spectral radius or Eigen value of $\mathcal{F}\mathcal{V}^{-1}$ is the required smoking generation number obtained by $R_s = \rho(\mathcal{F}v^{-1}) = \frac{\beta\alpha_1}{(\xi+\mu+\gamma)(\mu+\alpha_1)}$. Thus we can observe from the derived smoking generation number if $\beta\alpha_1 > (\xi+\mu+\gamma)(\mu+\alpha_1)$which makes $R_s > 1$ implies that there exists a widespread of the disease and if $\beta\alpha_1 < (\xi+\mu+\gamma)(\mu+\alpha_1)$ that is $R_s < 1$ which implies that the disease dies out.

Positivity of the Solutions

Theorem 1

All solutions of the differential equations of the dynamical systems (5)-(9) are positive.

Proof

i. The solution of the differential equation (2*) with initial condition $P(0) = P_0, P(0) > 0$ is

$$P(t) = p(0)e^{-\beta G(t)+\beta G(0)-\mu t} + \int_0^t e^{\mu(s-t)+\beta(G(s)-G(t))}\mu_0 N(s)ds > 0 \text{ for all } t > 0.$$

ii. The solution of the system (3*) with initial condition $E(0) = E_0$, $E(0) > 0$is

$$E(t) = E(0)e^{-(\mu+\alpha_1)t} + \int_0^t e^{(\mu+\alpha_1)(s-t)}\left(\frac{\beta P(s)S(s)}{N(s)}\right)ds > 0 \; for\; all\; t > 0.$$

iii. The solution of the system (4*) with initial condition $S(0) = S_0, S(0) > 0$is

$$S(t) = S(0)e^{-(\mu+\gamma+\xi)t} + \int_0^t e^{(\mu+\gamma+\xi)(s-t)}\big(\alpha_1 E(s) + \alpha Q(s)\big)ds > 0 \; for\; all\; t > 0.$$

iv. The solution of the system (5*) with initial condition $Q(0) = Q_0, Q(0) > 0$ is

$$Q(t) = Q(0)e^{-(\alpha+k+\mu)t} + \int_0^t e^{(\alpha+k+\mu)(s-t)}\big(\gamma\sigma S(s)\big)ds > 0 \; for\; all\; t > 0$$

v. The solution of the system (6*) with initial condition $R(0) = R_0$, $R(0) > 0$is

$$R(t) = R(0)e^{-\mu t} + \int_0^t e^{\mu(s-t)}(\, kQ(s) + \gamma(1-\sigma)S(s))ds > 0 \; for\; all\; t > 0$$

Hence we conclude that all solutions of the differential equations of (5)-(9) are positive.

Boundedness of the Solutions

Theorem 2

All solutions of the differential equations of the dynamical system (5)-(9) are bounded for all $t > 0$.

Proof

Let us assume that $\mu_0 < \mu$ with initial condition $N(0) = N_0$. After certain simplification in the solution of $\frac{dN(t)}{dt} = \frac{dP(t)}{dt} + \frac{dE(t)}{dt} + \frac{dS(t)}{dt} + \frac{dQ(t)}{dt} + \frac{dR(t)}{dt}$ we do have $0 < N(t) \le N_0 e^{-rt}$. Thus this shows that all solutions of the differential equations of the dynamical system (5)-(9) are bounded$t > 0$.

Equilibrium Points and Their Stability Analysis

Scaling:Let us represent the proportion of the compartments $P(t), E(t), S(t), Q(t)$ and$R(t)$ by:

$p(t) = \frac{P(t)}{N(t)}$, $e(t) = \frac{E(t)}{N(t)}$, $s(t) = \frac{S(t)}{N(t)}$,$q(t) = \frac{Q(t)}{N(t)}$ and$r(t) = \frac{R(t)}{N(t)}$where $p(t) + E(t) + s(t) + q(t) + r(t) = 1$. With this representation we do have a new dynamical system which is equivalent to the dynamical system (5)-(9)

$$\frac{dp}{dt} = \mu_0 - \beta p(t)e(t) - \mu p(t) \qquad (10)$$

$$\frac{de}{dt} = \beta p(t)s(t) - \mu e(t) - \alpha_1 e(t) \qquad (11)$$

$$\frac{ds}{dt} = \alpha_1 e(t) + \alpha q(t) - (\mu + \gamma + \xi)s(t) \qquad (12)$$

$$\frac{dq}{dt} = -(\alpha + k + \mu)q(t) + \gamma\sigma s(t) \qquad (13)$$

$$\frac{dr}{dt} = kq(t) + \gamma(1-\sigma)s(t) - \mu r(t) \qquad (14)$$

The Equilibrium Points of the System

The equilibrium points of the dynamical system (10)-(14) are obtained by making $\frac{dp}{dt} = \frac{de}{dt} = \frac{ds}{dt} = \frac{dq}{dt} = \frac{dr}{dt} = 0$. The Smoking free equilibrium point is determined by the assumption $e = s = q = r = 0$and $\mu_0 \ne \mu$. With this assumption the smoking free equilibrium point is $(p, e, s, q, r) = (\frac{\mu_0}{\mu}, 0,0,0,0)$. The Smoking present equilibrium point is determined by the assumption $s \ne 0$ is

$$(p, e, s, q, r) = \Big(\frac{\eta\mu_0}{\psi\beta R_s}, \frac{\psi R_s}{\eta} - \frac{\mu}{\beta}, \frac{\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta} - \frac{\mu}{\beta}\right), \frac{\gamma\sigma\alpha_1}{\eta(\mu+\gamma+\xi)}\Big(\frac{\psi R_s}{\eta} - \mu\beta, \psi Rs\eta - \mu\beta\gamma\zeta\mu\eta\mu + \gamma + \xi)$$

where $\psi = \mu_0(\mu + \alpha + \kappa), \eta = (\mu + \alpha + \kappa) - \frac{\alpha\gamma\sigma}{(\mu+\gamma+\xi)}$and $\zeta = \frac{k\sigma\alpha_1}{1} + \frac{(1-\sigma)\alpha_1\psi}{\mu_0}$.

Stability Analysis of the smoking free equilibrium point$(p, e, s, q, r) = (\frac{\mu_0}{\mu}, 0, 0, 0, 0)$

Then the Jacobean matrix at the smoking free equilibrium point $(\frac{\mu_0}{\mu}, 0,0,0,0)$ is

$$J\left(\frac{\mu_0}{\mu},0,0,0,0\right)=\begin{pmatrix}-\mu & \frac{\beta\mu_0}{\mu} & 0 & 0 & 0\\ 0 & -\mu-\alpha_1 & \frac{\beta\mu_0}{\mu} & 0 & 0\\ 0 & \alpha_1 & -\mu-\gamma-\xi & \alpha & 0\\ 0 & 0 & \gamma\sigma & -\alpha-k-\mu & 0\\ 0 & 0 & \gamma(1-\sigma) & k & -\mu\end{pmatrix}$$

The corresponding characteristic equation is $(-\mu-\lambda)^2[\lambda^3+a_1\lambda^2+a_2\lambda+a_3]=0$ where $a_1=(3\mu+\alpha_1+\gamma+\xi+\alpha+k)$, $a_2=(\mu+\alpha_1)(\mu+\gamma+\xi)+(\alpha+k+\mu)(2\mu+\alpha_1+\gamma+\xi)-\alpha\gamma\sigma-\frac{\beta\mu_0}{\mu}\alpha_1$ and $a_3=(\mu+\alpha_1)(\mu+\gamma+\xi))(\alpha+k+\mu)-\alpha\gamma\sigma\mu-\alpha\gamma\sigma\alpha_1-\frac{\beta\mu_0}{\mu}\alpha_1(\alpha+k+\mu)$.

The Routh-Hurwitz stability criterion states that if we obtain all the algebraic sign of the first column have the same sign which implies that all the roots have negative algebraic sign and this shows that the smoking free equilibrium point is stable (there is no smoking).

Stability Analysis of the Smoking Present Equilibrium Point

$$(p,e,s,q,r)=\left(\frac{\eta\mu_0}{\psi\beta R_s},\frac{\psi R_s}{\eta}-\frac{\mu}{\beta},\frac{\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right),\frac{\gamma\sigma\alpha_1}{\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right),\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)\frac{\gamma\zeta}{\mu\eta(\mu+\gamma+\xi)}\right)$$

The Jacobean matrix at the smoking present equilibrium point is

$$J(p,e,s,q,r)=\begin{pmatrix}-\frac{\psi\beta R_s}{\eta} & -\frac{\eta\mu_0}{\psi R_s} & 0 & 0 & 0\\ \frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right) & -\mu-\alpha_1 & \frac{\eta\mu_0}{\psi R_s} & 0 & 0\\ 0 & \alpha_1 & -\mu-\gamma-\xi & \alpha & 0\\ 0 & 0 & \gamma\sigma & -k-\mu-\alpha & 0\\ 0 & 0 & \gamma(1-\sigma) & k & -\mu\end{pmatrix}$$

Where $F=-\frac{\psi\beta R_s}{\eta}$, $H=\frac{\eta\mu_0}{\psi R_s}$, $G=\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)$

The corresponding characteristic equation is $\lambda^5+a_1\lambda^4+a_2\lambda^3+a_3\lambda^2+a_4\lambda+a_5=0$ where

$$a_1=(k+\mu+2\alpha+\gamma+\xi)-\frac{\mu_0(\mu+\alpha+\kappa)\beta R_s}{(\mu+\alpha+\kappa)-\frac{\alpha\gamma\sigma}{(\mu+\gamma+\xi)}}$$

$$\begin{aligned}a_2={}&(2\mu+\alpha+\gamma+\xi)(k+\alpha)+(\mu+\gamma+\xi)(\mu+\alpha)+\frac{\eta\mu_0}{\psi R_s}\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)\\&-\left(\left(-\frac{\psi\beta R_s}{\eta}+\mu\right)(\mathrm{k}+2\mu+2\alpha+\gamma+\xi)-\mu(k+\mu+\alpha)-\alpha\gamma\sigma-\frac{\eta\mu_0}{\psi R_s}\alpha_1\right)\end{aligned}$$

$$\begin{aligned}a_3={}&\left(-\frac{\psi\beta R_s}{\eta}+\mu\right)\left(\mu k+\mu^2+\alpha\mu-(2\mu+\alpha+\gamma+\xi)(k+\alpha)-(\mu+\alpha)(\mu+\gamma+\xi)+\alpha\gamma\sigma+\alpha_1\frac{\eta\mu_0}{\psi R_s}\right)\\&-((k+\mu+\alpha)\mu(2\mu+\alpha+\gamma+\xi)+(\mu+\alpha)(\mu+\gamma+\xi)(k+\alpha)+\alpha^2\gamma\sigma\\&+\left(\left(\frac{\eta\mu_0}{\psi R_s}\right)\alpha_1(k+\alpha)-\mu\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)\frac{\eta\mu_0}{\psi R_s}\right.\\&\left.-\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)\frac{\eta\mu_0}{\psi R_s}(2\mu+\gamma+k+\alpha+\xi)\right)\end{aligned}$$

$$\begin{aligned}a_4={}&\left(-(2\mu^2+\mu\alpha+\mu\gamma+\mu\xi)(k+\mu+\alpha)+(\mu+\alpha)(\mu+\gamma+\xi)(k+\alpha)+\alpha^2\gamma\sigma-\frac{\eta\mu_0}{\psi R_s}\alpha_1(k\right.\\&\left.+\alpha)\right)\left(-\frac{\psi\beta R_s}{\eta}+\mu\right)-(\mu+\alpha)(\mu+\gamma+\xi)(k+\mu+\alpha)\mu-\alpha\gamma\sigma\mu(\mu+\alpha)\\&-\left(\frac{\eta\mu_0}{\psi R_s}\alpha_1\mu(k+\mu+\alpha)+\mu\frac{\eta\mu_0}{\psi R_s}\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)(2\mu+\gamma+\xi+k+\alpha)\right)\\&-\left(\frac{\eta\mu_0}{\psi R_s}\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)(\mu+\gamma+\xi)(k+\mu+\alpha)\right.\\&\left.-\alpha\gamma\sigma\frac{\eta\mu_0}{\psi R_s}\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta}-\frac{\mu}{\beta}\right)\right)\end{aligned}$$

$$a_5 = \left(-\frac{\psi\beta R_s}{\eta} + \mu\right)\Big(\mu(\mu+\alpha)(\mu+\gamma+\xi)(k+\mu+\alpha) + \alpha\gamma\sigma\mu(\mu+\alpha) + \mu\alpha_1(k+\mu+\alpha)(\frac{\eta\mu_0}{\psi R_s})\Big)$$
$$- (\mu\frac{\eta\mu_0}{\psi R_s}\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta} - \frac{\mu}{\beta}\right)(\mu+\gamma)(k+\mu+\alpha)$$
$$+ \mu\alpha\gamma\sigma(\frac{\beta\alpha_1\psi}{\mu_0\eta(\mu+\gamma+\xi)}\left(\frac{\psi R_s}{\eta} - \frac{\mu}{\beta}\right)))$$

For $n = 5$ if $a_1a_2a_3 > {a_3}^2 + {a_1}^2a_4(a_1a_4 - a_5)$, $(a_1a_2a_3 - {a_3}^2 - {a_1}^2a_4) > a_5(a_1a_2 - a_3)^2 + a_1(a_5)^2$.

The Routh-Hurwitz stability criteria ensures that if all of the all five Eigen values found from the above characteristic polynomial have negative real parts then the smoking present

Equilibrium point is stable. We note that a_i depend on the value of all the individual and parameter are nonnegative for $t \geq 0$.

Real Parameter Estimation and Numerical Simulation

Real Parameter Estimation

The considered mathematical models can be used to represent the dynamics of tobacco use in real life if we have reasonable parametric values. To determine real parametric values we have to consider some facts. When a person first becomes a smoker it is not likely that she/he quits for several years since tobacco contains nicotine, which is shown to be an addictive drug [2].

Our real data collected from our research site Haremaya town, Ethiopia is presented as follows:

Table 2. *This table shows the total population of Haremaya town*

No.	List of division	Population in each division
1.	Total population in Haremaya town	15317
2.	Number of men in Haremaya town	7796
3.	Number of women in Haremaya town	7521
4.	Total people in sample taking area	5070
5.	Number of men in sample taking area	2558
6.	Number of women in sample taking area	2512

Table 3. *This table shows number of individuals in the Nonsmoker, Exposed, Smoker, Temporarily quit smokers and permanently quit smokers.*

No.	Compartments	Symbol	Total number
1.	Nonsmoker	P	30
2.	Exposed	E	20
3.	Smoker	S	48
4.	Temporarily quit smokers	Q	16
5.	Permanently quit smokers	R	6
6.	Total people asked	N	120

Table 4. *This table shows parameter estimation based on the data obtained by Table 3*

No	Parameter	values	Description
1.	β	0.6234	Rate of transmission nonsmoker into exposed class(contact rate)
2.	γ	0.1833	Removal rate(rate of quitting smoking)
3.	α_1	0.3984	Rate of conversion exposed into smoker class
4.	α	0.533	Rate of return from temporarily quit smoking to smoker class
5.	K	0.05	Rate of removal from Q to R
6.	δ	0.7272	The probability of conversion into Q and R
7.	μ_0	0.03370	Birth rate
8.	μ	0.162	Natural death rate
9.	ξ	0.1805	Smoking cause death rate
10.	$\gamma\delta$	0.1333	The rate probability convert into temporarily quit smoking
11.	$\gamma(1-\delta)$	0.050	The rate probability convert into permanently quit smoking

The Smoking Generation Number

In this study we consider the quantity R_s which is known as the smoking generation number. This quantity measures the average number of new smokers generated by a single smoker in a population of potential smokers.

If $R_s < 1$ then the total number of smokers in the population can be reduced to zero.

That is a small inflow of smokers into the community will not generate large number of smokers but if $R_s > 1$ the total number of smokers in the population increases that is a small inflow of smokers into the community will generate large number of smokers. In this model based on the real data parametric estimation we do have $R_s = \frac{\beta\alpha_1}{(\xi+\mu+\gamma)(\mu+\alpha_1)} = 1.04184235$. That is we do have $R_s > 1$ which implies that large number of smokers exists in the community of Haremaya town.

Stability Analysis on the Smoking Free Equilibrium Point $\left(\frac{\mu_0}{\mu}, 0, 0, 0, 0\right) = (0.2247, 0, 0, 0, 0)$

The Jacobian matrix at the smoking free equilibrium point is

$$\boldsymbol{J}(0.2247,0,0,0,0) = \begin{pmatrix} -0.15 & 0.146033 & 0 & 0 & 0 \\ 0 & -0.7166 & 0.146033 & 0 & 0 \\ 0 & 0.5666 & -0.3333 & 0.533 & 0 \\ 0 & 0 & 0.1333 & -0.733 & 0 \\ 0 & 0 & 0.05 & 0.05 & -0.15 \end{pmatrix}$$

The corresponding characteristic polynomial is $\lambda^5 + 2.0829\lambda^4 + 1.4120\lambda^3 + 0.3600\lambda^2 + 0.0383\lambda + 0.0014 = 0$. Applying the Routh-Hurwitz stability criterion we do have the array

λ^5	1	1.4120	0.0383	0
λ^4	2.0829	1.3239	0.0047	0
λ^3	1.1862	0.0867	0	0
λ^2	1.1600	0.0047	0	0
λ^1	0.0950	0	0	0
λ^0	0.0047	0	0	0

This shows that all the elements in the first column have the same sign thus the roots of the characteristic degree polynomial are all negative. This implies that the smoking free equilibrium point (0.2247,0,0,0,0)is stable.

Stability Analysis on the Smoking Present Equilibrium Point (0.8017, -0.1661, -0.2372, -0.0406, -0.4163)

Then the Jacobean matrix at the smoking present equilibrium point is

$$\begin{pmatrix} -0.042033 & -0.521134 & 0 & 0 & 0 \\ -0.154313 & -0.7166 & 0.521134 & 0 & 0 \\ 0 & 0.566 & -0.4933 & 0.533 & 0 \\ 0 & 0 & 0.1333 & -0.733 & 0 \\ 0 & 0 & 0.05 & 0.05 & -0.15 \end{pmatrix}$$

The corresponding characteristic equation is

$$\lambda^5 + 2.1349\lambda^4 + 1.1733\lambda^3 + 0.0614\lambda^2 - 0.0342\lambda - 0.0036 = 0$$

Applying the Routh-Hurwitz stability criterion we do have the array

λ^5	**1**	1.1733	−0.0342	0
λ^4	2.1349	0.0614	−0.0036	0
λ^3	1.1445	−0.0358	0	0
λ^2	0.1469	0.0036	0	0
λ^1	−0.2203	0	0	0
λ^0	0.0036	0	0	0

This shows that all the element of the first column has not the same sign therefore the smoking present equilibrium point is unstable.

NUMERICAL ANALYSIS

The numerical analysis is obtained from the graphs of the smoking generation number with respect to the real parameters obtained and given in Table 4.

In this part we can determine which parameter is sensitive in the smoking dynamics.

Let us consider the control parameter to be the rate of transmission of nonsmoker into exposed class (contact rate) β and the remaining parameters are constant.

The graphical representation of the smoking generation number versus the contact rate β is obtained by.

Rate of Transmission of Nonsmoker into Exposed Class (Contact Rate) Β

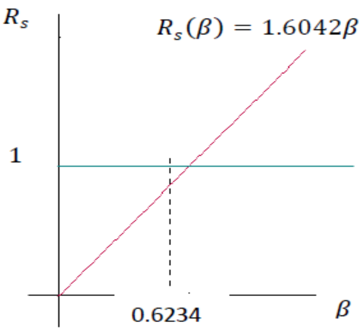

Figure 3. *From this graph we can observe that when the transmission rate $\beta > 0.6234$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the contact rate β exceeds 0.6234.*

Rate of Conversion Exposed into Smoker Class α_1

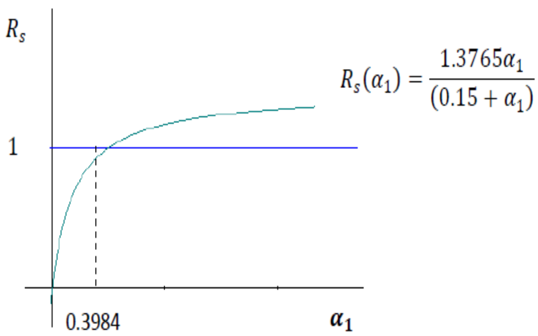

Figure 4. *From this graph we can observe that when the conversion rate $\alpha_1 > 0.3984$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the conversion rate α_1 exceeds 0.3984.*

Let us consider the control parameter to be α_1 which is the rate of conversion is from exposed into smoker class and the remaining parameters are constant. The graphical representation of the smoking generation number versus the conversion rate α_1 is obtained by.

Smoking Cause Death Rate ξ

Let us consider the control parameter to be ξ which the Smoking cause death rate and the remaining parameters are constant. The graphical representation of the smoking generation number versus the smoking cause death rate ξ is obtained by.

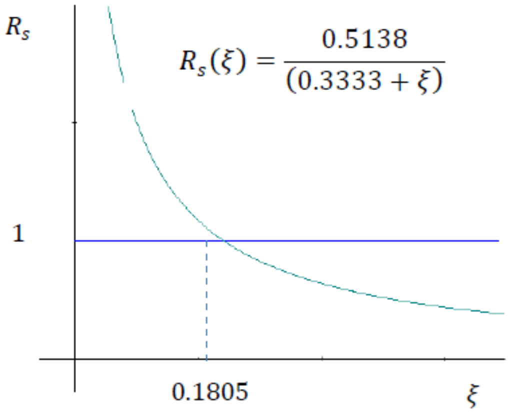

Figure 5. *From this graph we can observe that when smoking death rate $\xi > 0.1805$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the smoking death rate ξ exceeds 0.1805.*

Natural Death Rate μ

Let us consider the control parameter to be μ which is the Natural death rate and the remaining parameters are constant. The graphical representation of the smoking generation number versus the natural death rate μ is obtained by.

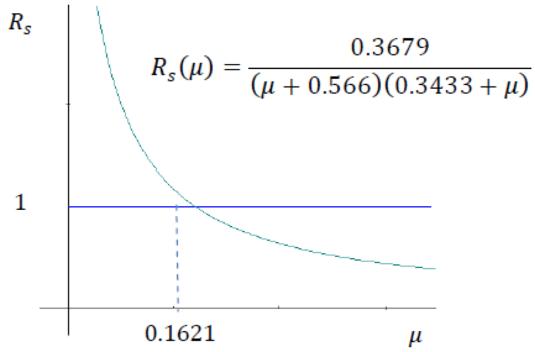

Figure 6. *From this graph we can observe that when the natural death rate $\mu > 0.1621$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the natural death rate μ exceeds 0.1621.*

Rate of Removal from Smoker to Temporarily Quit and Permanent Quit Smokers γ

Let us consider the control parameter to be γ which is the Rate of removal from smoker to temporarily quit and permanent quit smokers and the remaining parameters are constant. The graphical representation of the smoking generation number versus the Rate of removal from smoker to temporarily quit and permanent quit smokers γ is obtained by.

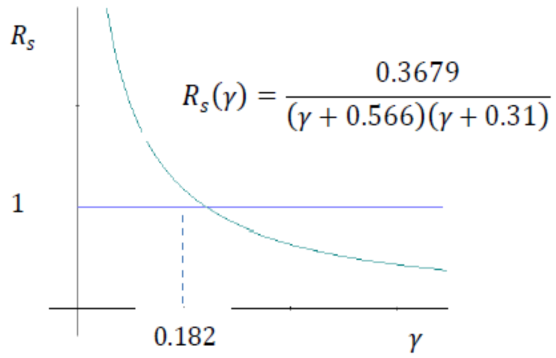

Figure 7. *From this graph we can observe that when the rate of removal from smoker to temporarily quit and permanent quit smokers is $\gamma > 0.182$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the Rate of removal from smoker to temporarily quit and permanent quit smokers γ exceeds 0.182.*

Discussions and Results

We considered systems of nonlinear differential equations which are used to study the dynamics of tobacco smoking in the case of Haremaya town Ethiopia. By adopting and extending an appropriate mathematical model on the dynamics of tobacco smoking we found that the number of cases one case generates on average over the course of its infectious period which is known as a smoking generation number R_s which determines how to spread and control tobacco smoking activities in the community.

In the O. Sharomi and A.B.Gumel mathematical model the authors divided the community into four compartments namely, the nonsmoker class $P(t)$, the smoker class $S(t)$, the temporarily quit smoking class $Q(t)$ and the permanent quit smoking class $R(t)$. To study the dynamics of tobacco smoking in our case we extend the O.Sharomi and A.B.Gumel mathematical model by adding one more compartment which we call it the exposed class $E(t)$ in between the non smoker class $P(t)$ and the smoker class $S(t)$ and by adding one parameter smoking cause death rate in the smoking class. In the O.Sharomi and A.B.Gumel mathematical model the authors assumed that the natural birth rate and natural death rate are the same but in our extended model we assumed that the natural birth rate and natural death rate are different.

Using second generation matrix we derived the formula for the smoking generation number of the extended model $R_s = \frac{\beta\alpha_1}{(\xi+\mu+\gamma)(\mu+\alpha_1)}$ which depends on our parameters. Using real data collected from Haremaya town we set whether the smoking generation number is greater than or less than unity. If the smoking generation number is greater than unity then smoking habit in the community spreads very high but if the smoking generation number is less than unity then the smoking habit in the community is low. Thus from this formula we can observe that the smoking generation number depends on five parameters namely: the rate of transmission from nonsmoker into exposed class β, the rate of conversion from exposed class into the smoker class α_1, the smoking cause death rate ξ, the natural death rate μ and rate of quit smoking temporarily or permanently γ.

The numerical analysis based on the real data obtained in figure 3 – 7 we found that: when the transmission rate $\beta > 0.6234$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the contact rate β exceeds 0.6234, when the conversion rate $\alpha_1 > 0.3984$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the conversion rate α_1 exceeds 0.3984, when smoking death rate $\xi > 0.1805$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the smoking death rate ξ exceeds 0.1805, when the natural death rate $\mu > 0.1621$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the natural death rate μ exceeds 0.1621 and when the Rate of removal from smoker to temporarily quit and permanent quit smokers is $\gamma > 0.182$ then the reproduction number $R_s > 1$. This means that the smoking habit spreads in the society when the Rate of removal from smoker to temporarily quit and permanent quit smokers γ exceeds 0.182.

Conclusions and Recommendations

Based on our real data collected from our research site we have the reproduction number is $R_s = 1.04184235$. This shows that the smoking generation number is greater than unity and thus the smoking habit in Haremaya town is very high and then this shows that the number of smokers in the community increases. As this smoking habit is very dangerous to the youngsters who are productive in the economic activity of the nation, therefore we recommend the following suggestions to reduce the number of smokers in the community.

To decrease the number of smokers we must control the smoking generation number which is the number of cases one case generates on average over the course of its infectious period should be less than unity. For this we must control two parameters namely the contact rates β not to exceed 0.6234, the conversion rate $\alpha_1$not to exceed 0.3984, smoking death rate ξ not to less than 0.1805, the natural death rate μ not to less than 0.1621and the rate of removal from smoker to temporarily quit and permanent quit smokers γ not to be less than 0.182.

From our observation the main factors of the existence of the above five parameters are peer pressure, smoking by parents around home and smoking around public service area. Therefore to reduce the number of smokers in Haremaya town we recommend the following. Complete bans or enforce bans on tobacco advertising, promotion and sponsorship, offer help to quit tobacco use, warn people about the dangers of tobacco, making tobacco smoking control policy that smoking is prohibited in public area. In this work we did not considered factors like sex structure, age structure and ethnicity therefore the coming researchers better to add such factors.

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