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Spread and Control of Hiv/Aids in the Case of Sheno Town Ethiopia: A Mathematical Model Analysis

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ABSTRACT

In this work we have considered nonlinear differential equations in four dimensional dynamical systems. We divide the whole population of Sheno town into four compartments: The susceptible class S, The unaware class I_1, The aware class I_2 and the full-fledged AIDS class A to study the spread and control of HIV/AIDS in the case of Sheno town, North Shoa zone, Ethiopia. We found that the basic reproduction number $R_0 = \frac{\beta_1+\varepsilon_1}{\theta+\mu+\delta_1+1-\varepsilon_1}$ which depends on five parameters. We also found that the numerical value of the reproduction number based on the real data collected from Sheno town is $R_0 = 0.7464 < 1$. This in principle shows that the HIV/AIDS disease spreads slowly in the community of Sheno town. We investigate two equilibrium points namely stable diseases free equilibrium point and unstable endemic equilibrium point. To control the spread of HIV/AIDS we then identify the control parameter which gives insight to decrease or stop its spread. The basic control parameters that can decrease the spread of the disease are $\beta_1 = 0.0214$. This is the rate of transmission from susceptible class into infected unaware class.Therefore to keep the basic reproduction number is less than one the parameter β_1 should be less than 0.0214. The effects of the remaining parameters on the basic reproduction number are discussed in detail in their subsections.

Keywords: *Nonlinear differential equations, Stability analysis, Reproduction number, Control Parameters*

INTRODUCTION

The main use of mathematics in epidemiology is to gain insight on epidemics, to see how the dynamic of an infectious disease depends on the basic parameters that characterize it. One of the greatest causes of mortality in the sub-Saharan Africa, particularly among young adults is HIV/AIDS. Many mathematical models have been suggested for describing the epidemiology as well as the epidemiological consequences of the epidemic [4]. Human Immunodeficiency Virus, HIV is a lent virus that causes Acquired Immune Deficiency Syndrome, AIDS. The report of WHO, World Health Organization state that HIV infection in humans is considered as a pandemic disease. In the year 1981 till 2006 the records show that more than 25million people have been killed worldwide by HIV/AIDS. HIV infected about 90 million people in African continent[1]. HIV infection transforms from an infected person to victimif the bodily fluids of someone who already has HIV get into the other through blood transfusion, by unprotected sexual intercourse (heterosexual or homosexual), by vertical transmission (mother to child), sharing needles and syringes. Migration and immigration of the people from one country to another country due to different reasons play a crucial role in the evolution and spread of HIV/AIDS epidemic [2].The study of HIV transmission and the dynamics of the disease have been of a great interest to both applied mathematicians and Biologists. Mathematical modeling has proved to be an important tool in analyzing the spread and control of HIV disease [5, 6].The results of modeling and analysis help to improve understanding of the major contributing factors to the pandemic. Mathematical models have been studied and important inferences have been drawn in case of epidemics like Ebola, Breast cancer, Malaria, Tuberculosis and Influenza [7, 8].Several researchers have developed HIV/AIDS models so as to understand and explain the dynamics and the spread of the disease and succeeded to a large extent. A theoretical framework describing the transmission of HIV/AIDS with screening of unaware infective persons is presented in [3, 9]. In this work we by adopting the basic SIR

(Susceptible – Infected – Recovered) model and we extend into S$I_1 I_2$A where the infected I class again categorized into unaware class I_1 and aware class I_2. The recovered class in the basic model replaced by full-fledged aids class A as there is no recovery individuals from the HIV/AIDS disease.

Statement of the Research Problem

In this study we raised the following research problems that help us to investigate the control parameter of the spreading of HIV/AIDS: What are the equilibrium points with their stability analysis of the dynamical system? What are the major factors/parameters that affect the control of HIV/AIDS? What is the basic reproduction number R_0?What are the most influential control parameters that affect the basic reproduction number?

Objectives

The main objective of this work is to develop a mathematical model for analyzing the dynamics of HIV/AIDS in the case of Sheno town.

The specific objectives are: developing an appropriate mathematical model for HIV/AIDS, analyzing the stability of the equilibrium points of the mathematical model, identifying the basic reproductive number, determining the most influential control parameter/s.

Methodology

We used different kinds of methodologies used for nonlinear dynamical system analysis: a method for determining equilibrium points of the nonlinear dynamical system, Routh- Hurwitz stability criterion to determine the stability of the investigated equilibrium points, a second generation matrix to derive the basic reproduction number.

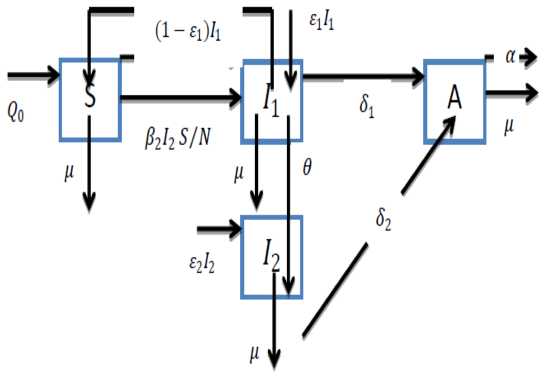
THE MATHEMATICAL MODEL

State Variables and Model Assumptions

Let the state variables S, I_1 ,I_2 and A represents respectively susceptible class, disease unaware class, disease aware class and full-fledged class. We assumed:the newly requirement rate and natural death rates are equal, the population under our work is heterogeneous, the whole human population is divided in to four classes, the HIV can transmitted by the heterosexual intercourse with infective peoples and mother to child (vertical) transmission, the full-fledged AIDS class is sexually inactive, all the new infected people are assumed to be initially unaware of the infection, the probability of transferring the disease tosusceptible population by unaware infected person is more than by aware infected person $\beta_1 > \beta_2$ and the unaware infected people grow to AIDS much faster than the aware infected people i.. e. $\delta_1 > \delta_2$.

Flow chart of the Mathematical Model

Based on the above state variables and model assumptions we develop the following flow chart of the dynamical system



The corresponding dynamical system is

$$\frac{dS}{dt} = Q_0 - \left(\frac{\beta_1 I_1 + \beta_2 I_2}{N}\right) s - \mu s + (1-\varepsilon_1) I_1 s$$

$$\frac{dI_1}{dt} = \left(\frac{\beta_1 I_1 + \beta_2 I_2}{N}\right) s + \varepsilon_1 I_1 - (\theta + \delta_1 + \mu) I_1 - (1-\varepsilon_1) I_1$$

$$\frac{dI_2}{dt} = \varepsilon_2 I_2 + \theta I_1 - (\delta_2 + \mu) I_2$$

$$\frac{dA}{dt} = \delta_1 I_1 + \delta_2 I_2 - (\alpha + \mu) A$$

This is a system of nonlinear ordinary differential equation with initial conditions with initial conditions $S(0) = S_0$,$I_1(0) = I_{1\,0}$,,$I_2(0) = I_{2\,0}$,$A(0) = A_0$

Positivity of Solutions

Theorem

The solutions of all nonlinear differential equations in the dynamical system are non-negative.

Proof:

Solving analytically all the state variables we found that their respective solutions are

$$S(t) = S_0 e^{-Q(t)+Q_o-\mu t}\int_0^t Q_o e^{Q(s)-Q(t)+\mu(S-t)}\,ds > 0$$

$$I_1(t) = I_1(o)e^{-kt+\beta_1 Q(t)-\beta_1 Q(o)} + e^{-kt+\beta_1 Q(t)-\beta_1 Q(o)}\int_0^t (\beta_2 I_2\, S/N)e^{ks}\beta_1 Q(S) - \beta_1 Q(o)ds > 0$$

$$I_2(t) = I_2(o)e^{-ht} + e^{-ht}\int_0^t \theta I_1(S)e^{hs}ds > 0$$

$$A(t) = A_0 e^{-(\alpha+\mu)} + e^{-(\alpha+\mu)t}\int_0^t (\delta_1 I_1 + \delta_2 I_2)e^{(\alpha+\mu)s}ds > 0$$

Then the dynamical system is epidemiologically meaningful and well posed.

Boundedness of the Solution

Theorem

All solutions of the nonlinear ordinary differential equations in the dynamical system are bounded

Proof

$\frac{dN}{dt} = \frac{ds}{dt} + \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dA}{dt} = Q_o + \varepsilon_2 I_2 + [(1-\varepsilon_1)S + p_1]I_1 - \mu N - \alpha A$. The solution is obtained after some simple calculation and is

$N(t) \le [\left(\frac{Q_o}{\mu}\right) - N_o e^{-\mu t} \;\to 0 \;\; as \;\; t \to \infty$. And hence the solutions are bounded.

Equilibrium points of the Model

The equilibrium points of the dynamical system are obtained by making the right hand side of the system equal to zero. By this we found that two equilibrium points namely disease free equilibrium point E_o and endemic equilibrium point.

$$E_o = \left(\frac{Q_o}{\mu}, 0,0,0\right)$$

$$E^* = \left(\frac{\rho}{[\beta_1 - (1-\varepsilon_1)]\gamma_1 + \beta_2\gamma_2 + \mu}\right), \frac{\beta_1 + \varepsilon_1}{(\theta + \mu + \delta_1 + 1 - \varepsilon_1)}, \frac{\theta\gamma_1}{\delta_2 + \mu - \varepsilon_2}, \frac{\delta_1\gamma_1 + \delta_2\gamma_2}{\alpha + \mu})$$

Stability Analysis of the Equilibrium Point

In the dynamical systems of nonlinear differential equations the stability analyses of the existing equilibrium points play an important role in the analysis of the models. We use the method Routh-Hurwitz stability criterion to prove the stability of the disease free equilibrium point is stable and the endemic equilibrium point is unstable which is also guaranteed by the real data analysis of basic reproduction number $R_0 = \frac{\beta_1+\varepsilon_1}{\theta+\mu+\delta_1+1-\varepsilon_1} = 0.7464 < 1$ which is shown in the next section.

Basic Reproduction Number R_o

The basic Reproduction number is the average number of secondary cases produced by a typical infected individual during his or her entire life as infectious or infectious period when introduced or allowed to live in a population of susceptible. It is obtained by the method of second generation matrix and we found

$$R_0 = \frac{\beta_1 + \varepsilon_1}{(\theta + \mu + \delta_1 + 1 - \varepsilon_1)}$$

In principle if $R_0 < 1$ then the disease does not spread in the community or the disease dies out. But if $R_0 > 1$ then the disease spreads in the community.

Model Application Based on the Real Data Collected from Sheno Town

To determine the equilibrium points of the system and their stability analysis based on the real data collected from the research site we summarized the parameters found in the dynamical system by the table below.

Descriptions	Notations	Number
Total population	S	27300
Number of initial susceptible individuals	S_0	26130
Number of un aware infective individuals	I_1	585
Number of aware infective individuals	I_2	369
Number of HIV/AIDS individuals	A	216

Table 1: State variable description with real data

Descriptions	**Notations**	Number
Fraction of susceptible individuals	s_0	0.95714286
Fraction of unaware peoples	I_{1_0}	0.021428571
Fraction of aware peoples	I_{2_0}	0.013516483
Fraction of AIDS peoples	A_o	0.007912087

Table 2: Fraction of individuals based on real data

Parameters	Descriptions
β_1	transmission rate from susceptible individuals to unaware individuals
β_2	transmission rate from susceptible individuals to aware individuals
δ_1	transmission rate from unaware individuals to the aids class
δ_2	transmission rate from aware individuals to the aids class
ε_1	the rate of probabilities of newly born entered to unaware class
ε_2	the rate of probabilities of newly born entered to aware class
α	Aids disease cause death rate
μ	natural death rate
R_0	basic reproduction number
ρ	the number of susceptible at a constant rate
ϑ	the mean latency period of HIV/AIDS
φ	the mean infection period of HIV/AIDS
γ_1	the rate of unaware infected people
γ_2	the rate of aware infected people
θ	the rate of infective decreases from unaware to aware class

Table 3: Parameter descriptions

Latency Period and Incubation Period of AIDS

The latency period of HIV/AIDS is less than or equal to 90 days between sexual contact and symptom onset and thus

- The mean latency period of HIV/AIDS is $\frac{90+14}{2} = 52\ days$

The infectious period or incubation period of AIDS is 90 days to 900 days and thus

- The mean infection period is $\frac{90+900}{2} = 495\ days$

Real Parameter Estimation

$$s_0 = \frac{number\ of\ initial\ susceptible\ individuals}{total\ population} = \frac{26130}{27300} = 0.9571$$

$$I_1 = \frac{number\ of\ un\ aware\ infective\ people}{total\ population} = \frac{585}{27300} = 0.02142$$

$$I_2 = \frac{number\ of\ aware\ infective\ people}{total\ population} = \frac{369}{27300} = 0.01352$$

$$A = \frac{number\ of\ AIDS\ Class}{total\ population} = \frac{216}{27300} = 0.0079$$

$$\beta_1 = \frac{effective\ unaware\ contacts}{total\ contacs} = \frac{585}{27300} = 0.0214$$

$$\beta_2 = \frac{effective\ aware\ contacts}{total\ contacs} = \frac{369}{27300} = 0.0135$$

$$\vartheta = \frac{1}{\text{mean latency period}} = \frac{1}{52} = 0.01923$$

$$\varphi = \frac{1}{\text{mean infection period}} = \frac{1}{495} = 0.0020$$

$$\mu = \frac{natural\ death\ rate}{total\ population} = \frac{87}{27300} = 0.0032$$

$$\alpha = \frac{number\ of\ individuals\ dies\ by\ HIV/AIDS}{total\ number\ of\ HIV\ positive} = \frac{79}{954} = 0.0828$$

$$\theta = \frac{the\ number\ of\ infective\ decreases\ from\ unaware\ to\ aware}{total\ infective\ from\ HIV/AIDS} = \frac{222}{954} = 0.2327$$

$$\varepsilon_1 = \frac{number\ of\ un\ aware\ susceptible\ new\ \ born}{total\ number\ of\ infective\ pregnacy\ women} = \frac{7}{9} = 0.7778$$

$$\varepsilon_2 = \frac{number\ of aware\ newly\ born\ is\ susceptible}{total\ number\ of\ infective\ pregnacy\ women} = \frac{2}{9} = 0.2222$$

$$\delta_1 = \frac{number\ of\ unaware\ individuals\ infected\ to\ AIDS}{total\ number\ of\ HIV\ Positive} = \frac{585}{954} = 0.6132$$

$$\delta_2 = \frac{number\ of\ aware\ individuals\ infected\ to\ AIDS}{total\ number\ of\ HIV\ Positive} = \frac{369}{954} = 0.3868$$

$$\gamma_1 = \frac{number\ of\ unaware\ infected\ people}{total\ population} = \frac{954}{27300} = 0.0349$$

$$\gamma_2 = \frac{the\ number\ of\ aware\ infected\ people}{total\ population} = \frac{585}{27300} = 0.0214$$

$$\rho = \frac{the\ number\ of\ susceptible\ at\ a\ constant\ rate}{total\ population} = \frac{60}{27300} = 0.0022$$

Real Data Analysis on the Reproduction Number

Based on the above real data we found that

- $R_0 = \frac{\beta_1+\varepsilon_1}{\theta+\mu+\delta_1+1-\varepsilon_1} = 0.7464 < 1$. This in principle HIV/AIDS does not spread in the community of Sheno town.

Real Data Representation of the Existing Equilibrium Points

Based on the above real data we found that

- Disease free equilibrium point is $E_0 = (1,0,0,0)$
- Endemic equilibrium point is $E^* = (0.6206, 0.7464, 0.04851, 0.3456)$

Stability Analysis of the Equilibrium Points

Based on Routh-Hurwitz stability criterion we found that

- The disease free equilibrium point $E_0 = (1,0,0,0)$ is stable
- The endemic equilibrium point $E^* = (0.6206, 0.7464, 0.04851, 0.3456)$ is unstable.

Numerical Analysis

In this section we found the impact of our ontrol parameters on the basic reproduction number.

Let Us Take Our Control Parameter to be β_1.

The basic control parameters that can decrease the spread of the disease isβ_1 which is the rate of transmission from susceptible class into infected unaware class. The graphical representation of the control parameter $\beta_1$vs the basic reproduction number R_0 is given below

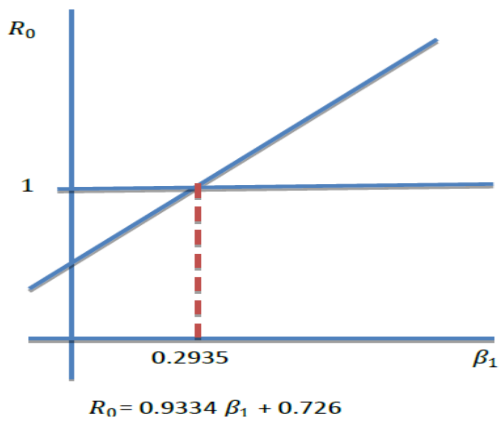
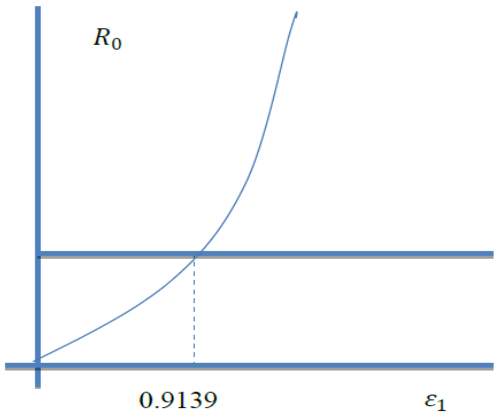


Figure 1. *This figure shows the effect of the control parameter β_1 on the basic reproduction number R_0. To control the spread of the HIV/AIDS, the numerical value of the control parameter β_1 never exceed 0.2935.*

Let Us Take Our Control Parameter to be ε_1.

The basic control parameters that can decrease the spread of the disease is β_1 which is the rate of probabilities of newly born entered to unaware class. The graphical representation of the control parameter $\varepsilon_1$vs the basic reproduction number R_0 is given below

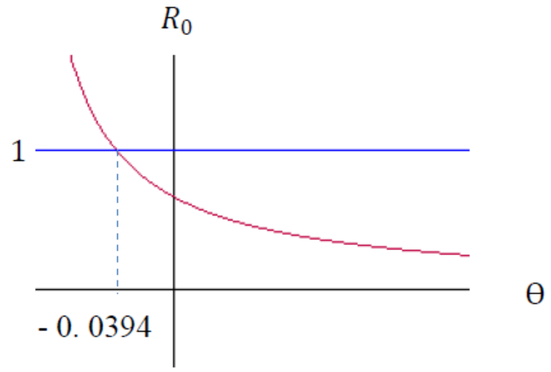


$$R_0 = \frac{0.0214+\varepsilon_1}{1.8491-\varepsilon_1}$$

Figure 2. *This figure shows the effect of the control parameter ε_1 on the basic reproduction number R_0. To control the spread of the HIV/AIDS, the numerical value of the control parameter ε_1 never exceed 0.9139.*

Let Us Take Our Control Parameter to be θ

The basic control parameters that can decrease the spread of the disease isθ which is the rate of infective decreases from unaware to aware class. The graphical representation of the control parameter $\varepsilon_1$vs the basic reproduction number R_0 is given below

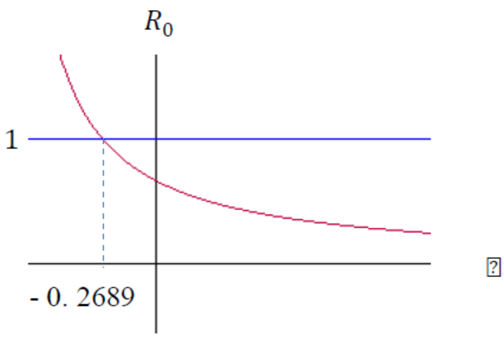


$$R_0 = \frac{07992}{\theta+08386}$$

Figure 3. *This figure shows the effect of the control parameter θ on the basic reproduction number R_0. To control the spread of the HIV/AIDS, the numerical value of the control parameter Θ never less than -0.0394.*

Let Us Take our Control Parameter to me µ

The basic control parameters that can decrease the spread of the disease isμ which is the natural death rate. The graphical representation of the control parameter μ vs the basic reproduction number R_0 is given below

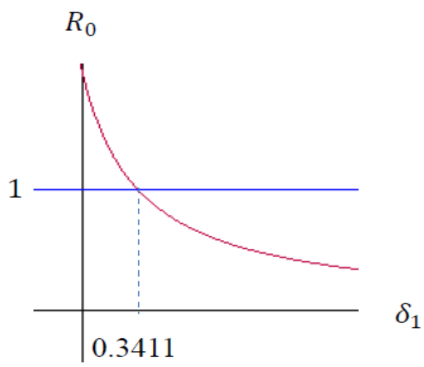


$$R_0 = \frac{07992}{⍰+1.0681}$$

Figure 4. *This figure shows the effect of the control parameter ⍰ on the basic reproduction number R_0. To control the spread of the HIV/AIDS, the numerical value of the control parameter ⍰ never less than -0.2689.*

Let Us Take Our Control Parameter to be δ_1

The basic control parameters that can decrease the spread of the disease isδ_1 which is the natural death rate. The graphical representation of the control parameter $\delta_1$vs the basic reproduction number R_0 is given below



$$R_0 = \frac{07992}{\delta_1+0.4581}$$

Figure 5. *This figure shows the effect of the control parameter δ_1 on the basic reproduction number R_0. To control the spread of the HIV/AIDS, the numerical value of the control parameter δ_1 never less than 0.3411.*

Conclusions And Recommendations

To control the spread of the HIV/AIDS we investigate three most influential control parameters to make the basic reproduction number R_0 to be less than one.

The numerical value of the control parameter β_1(the rate of transmission from susceptible class into infected unaware class) never exceed 0.2935, the numerical value of the control parameter ε_1 (the rate of probabilities of newly born entered to unaware class)never exceed 0.9139 and the numerical value of the control parameter δ_1 (the natural dease rate) never less than 0.3411.

References

[1] Berec, Lud'ek, 2011, Mathematical modeling in ecology and epidemiology. Masaryk university x. Xia. HIV/AIDS modeling and control, automatic, 2004

[2] Deterministic structured population epidemic models, Barbara Boldin, University of Ljubljana, Slovenia, 2003

[3] Johnson, Teri &Mc, Barry. Mathematical Modeling of Diseases: Susceptible- Infected – Recovered (SIK) Model. Quarries University, Minnesota, 2009.

[4] Kermack, W. O. and McKendrick, A. G. (1927). Contributions to the mathematical theory of epidemics. Proceedings of the Royal Society of London. Series A, volume 115,

[5] Mushanyu, P. Mafuta, 2014). Extension of the analysis of the MISR model by applying the laplace-adomian decomposition method. Nhawu University .On differentiability of nonlinear ODE Models and applications in viral Dynamics.https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3140286/by H MIAO - 2011

[6] Samuel okyere, EPIOEIOLOGICAL Model of Influenza (HINI) Transmission In Ashanti Region of Ghana, Master Thesis, June, 2012, Ghana.

[7] Waziri, Abdallah S &Massawe, Estomih S,(2012). Mathematical modeling of HIV/AIDS Dynamics with Treatment and vertical Transmission. University of Dares Salaam, Dares Salaam, Tanzania

[8] World health organization Regional office for the western pacific, (November 29-December 03,2010). Report workshop for research Design, methodology and proposal writing on infectious disease on poverty. Manila,

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