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Evaluation of Parameter Estimation Methods of Probability Distributions for Extreme Value Analysis of Temperature

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ABSTRACT

Extreme weather may cause substantial damage to the water resources systems through droughts, floods and ecological disturbances; and it may also affect human activities. Temperature is the prime parameter in the weather conditions of any region. In this context, analyzing of temperature events using probability distributions is identified as an effective tool for arriving at a specific value for design purposes. This paper illustrates the use of Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), Log Normal (LN2) and Log Pearson Type-3 (LP3) probability distributions in extreme value analysis of temperature data recorded at Hissar. Methods of Moments (MoM) and Maximum Likelihood Method (MLM) are used for determination of parameters of EV1, EV2, LN2 and LP3 distributions. The adequacy of fitting of probability distributions is evaluated by Goodness-of-Fit tests viz., Anderson-Darling and Kolmogorov-Smirnov and diagnostic test using D-index. The study showed that the LN2 (MLM) distribution is better suited probability distribution for estimation of absolute, dry bulb and wet bulb maximum temperature whereas EV1 (MLM) for estimation of absolute, dry bulb and wet bulb minimum temperature for Hissar.

Keywords: *Anderson-Darling test, D-index, Extreme Value Type-1, Kolmogorov-Smirnov Test, Log Normal, Maximum Likelihood Method, Temperature*

INTRODUCTION

Extreme weather conditions are reported to may cause substantial damage to the water resources systems through droughts, floods and ecological disturbances; and it may also affect human activities (Watson et al., 1996). The major parameter in the weather condition of a region is attributed to temperature. Research activities on extreme climate phenomena have directed towards their impacts by the effect of possible increases in frequency, duration and severity. Impacts of climate change would result from changes in variability and also the extreme event occurrence rather than from an increase in mean temperature (Parmesan et al., 2000; WMO, 2001; Hughes et al., 2007) to relatively small changes in the averages and variations of climate variables can induce considerable changes in the severity of extreme events (Colombo et al., 1999; Huth et al., 2000).

A number of probability distributions are available for Extreme Value Analysis (EVA) of temperature data. Out of which, Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), 2-parameter Log Normal (LN2) and Log Pearson Type-3 (LP3) are generally used for EVA of temperature (Vose et al., 2004; Cooley, 2005; Hasan and Yeong, 2014). Based on the theoretical applicability, parameter estimation procedures viz., Methods of Moments (MoM), Maximum Likelihood Method (MLM) and Order Statistics Approach (OSA) are used for determination of parameters (Bobee and Askhar, 1991). Generally, MoM is used in determining the parameters of the probability distributions. Sometimes, it is difficult to assess exact information about the shape of a distribution that is conveyed by its third and higher order moments. Also, when the sample size is small, the numerical values of sample moments can be very different from those of the probability distribution from which the sample was drawn. It is also reported that the estimated parameters of distributions fitted using MoM are often less accurate than those obtained by MLM. AERB (2008) guidelines suggested the 1000-year return period Mean+1σ (where Mean denotes the estimated temperature (x_T) and σ the Standard Error (SE)) value will be generally considered for arriving at a design value of maximum temperature whereas Mean-1σ value for minimum temperature. In the recent past, number of studies has been carried out by researchers adopting probability distributions for EVA of temperature.

Hughes et al. (2007) carried out statistical analysis using Generalized Extreme Value (GEV) distribution and time-series models for minimum and maximum temperatures in the Antarctic Peninsula. Hasan et al. (2013) applied GEV distribution for modelling the annual maximum temperature recorded at twenty-two meteorological stations in Malaysia. Vivekanandan (2015) applied EV1 and EV2 distributions for modelling of extreme rainfall, temperature and wind speed for Kanyakumari region. He found that the EV1 (OSA) is better suited probability distribution for modelling the series of annual extreme rainfall and annual maximum (or) minimum temperature whereas EV2 (OSA) for modelling the series of annual hourly maximum wind speed for Kanyakumari. Generally, when different probability distributions are used for EVA, a common problem that arises is how to determine which model fits best for a given set of data. This can be evaluated by quantitative assessment using Goodness-of-Fit (GoF) and diagnostic tests. GoF tests viz., Anderson-Darling (A^2) and Kolmogorov-Smirnov (KS) are applied for checking the adequacy of fitting probability distributions to the temperature data. A diagnostic test (using D-index) is applied for the selection of most suitable probability distribution for estimation of maximum or minimum temperature. This paper details the procedures involved in assessing the suitable probability distribution in modelling of temperature data (viz., absolute, dry bulb and wet bulb) though GoF and diagnostic tests for Hissar region.

METHODOLOGY

The effort made in this study is to assess the Probability Density Functions (PDFs) adopted in modelling of temperature data. For this, it is required to process and validate the data of the variate for application such as (i) perform statistical tests such as randomness, homogeneity and outliers; (ii) select the PDFs (EV1, EV2, LN2 and LP3) for modelling of temperature data; (iii) select parameter estimation methods (MoM and MLM); (iv) select GoF and diagnostic tests and (v) conduct the study and analyze the EVA results obtained thereof. Table 1 gives the PDFs with the corresponding quantile estimators (XT, also referred as Mean) of the distributions are presented in Table 1.

In Table 1, for LN2 distribution, the symbols α and β represents the mean and standard deviation of the log-transformed series of recorded data. α, β and γ denotes the location, scale and shape parameters of the distributions respectively. For EV1 and EV2 distributions, the reduced variate (Y_T) for a given return period (T) is defined by Y_T=-ln(-ln(1-(1/T))) while in the mathematical representation of LN2 and LP3, K_P denotes the frequency factor corresponding to the probability of exceedance. The Coefficient of Skewness (C_S) is C_S=0.0 for LN2 whereas C_S is based on the log transformed series of the recorded data for LP3. For the data series with AMINT, the value of Y_T will be read as Y_T=-ln(-ln(1/T)) and K_P is the frequency factor corresponding to the probability of non-exceedance for LN2 and LP3 distributions (Bobee and Askhar, 1991).

Table 1. *PDF and quantile estimator of probability distributions*

Distribution	PDF	Quantile estimator
EV1 (Location, Scale)	$f(x;\alpha,\beta)=\frac{e^{-(x-\alpha)/\beta}e^{-e^{-(x-\alpha)/\beta}}}{\beta}$, β>0	$x_T=\alpha+Y_T\beta$
EV2 (Scale, Shape)	$f(x;\beta,\gamma)=\frac{\gamma}{\beta}\left(\frac{\beta}{x}\right)^{\gamma+1}\exp\left(-\frac{x}{\beta}\right)^{-\gamma}$, β>0	$x_T=\beta\exp(Y_T/\gamma)$
LN2 (Location, Scale)	$f(x;\alpha,\beta)=\frac{1}{\beta x\sqrt{2\pi}}\exp\left(-\frac{(\ln(x)-\alpha)^2}{2\beta^2}\right)$, -∞<x<∞,β>0	$x_T=\exp(\alpha+K_P\beta)$
LP3 (Location, Scale, Shape)	$f(x;\alpha,\beta,\gamma)=\frac{1}{\beta x\Gamma\gamma}\left(\frac{\ln(x)-\alpha}{\beta}\right)^{\gamma-1}\exp\left(-\frac{(\ln(x)-\alpha)}{\beta}\right)$,β,γ>0	$x_T=\exp((\alpha+\beta\gamma)+K_P\beta\sqrt{\alpha})$

The detailed procedures adopted in estimation of maximum or minimum temperature using EV1, EV2, LN2 and LP3 distributions could be found in the text book of 'Flood frequency analysis' (Rao and Hameed, 2000). From the PDFs as given in Table 1, it could be noticed that the EV1 is double exponential function and EV2 is single exponential. Thus, the fitted curves are in linear form for EV1 and exponential form for EV2.

Goodness-of-Fit Tests

Generally, A^2 test is applied for checking the adequacy of fitting of EV1 and EV2 distributions. The procedures involved in application of A^2 test for LN2 and LP3 are more complex though the utility of the test is extended for checking the quantitative assessment. In view of the above, KS test is widely applied for the purpose of quantitative assessment. Theoretical descriptions of GoF tests are as follows:

A^2 statistic is defined as below:

$$A^2 = (-N) - (1/N)\sum_{i=1}^{N}\left\{\begin{matrix}(2i-1)\ln(Z_i) + \\ (2N+1-2i)\ln(1-Z_i)\end{matrix}\right\} \quad (1)$$

Here, $Z_i=F(x_i)$ for $i=1,2,3,\ldots,N$ with $x_1<x_2<\ldots<x_N$, $F(x_i)$ is the Cumulative Distribution Function (CDF) of i^{th} sample (x_i) and N is the sample size.

KS test statistic is defined as below:

$$KS = \underset{i=1}{\overset{N}{Max}}\left(F_e(x_i) - F_D(x_i)\right) \quad (2)$$

Here, $F_e(x_i)$ is the empirical CDF of x_i and $F_D(x_i)$ is the derived CDF of x_i by PDFs. In this study, Weibull plotting position formula is used for computation of empirical CDF (Cook, 2011). The theoretical values of A^2 and KS tests statistic for different sample size (N) at 1% significance level are available in the technical note on 'Goodness-of-Fit Tests for Statistical Distributions' by Charles Annis (2009).

Test criteria: If the computed value of GoF test statistic given by the distribution (or method) is less than that of theoretical value at the desired significance level then the distribution is assumed to be suitable for EVA at that level of significance.

Diagnostic Test

Sometimes the GoF test results would not offer a conclusive inference thereby posing a bottleneck to the user in selecting the suitable PDF for application. In such cases, a diagnostic test in adoption to GoF tests is applied. The selection of most suitable probability distribution for estimation of temperature could be performed through D-index test (USWRC, 1981), which is defined as below:

$$D-\text{index} = \left(1/\bar{x}\right)\sum_{i=1}^{6}\left|x_i - x_i^*\right| \quad (3)$$

Here, $\bar{x}$ is the average value of the annual maximum (or minimum) series of the recorded temperature data whereas x_i (i=1 to 6) and x_i^* are the six highest recorded and corresponding estimated values by different PDFs. The distribution has the least D-index would be considered as better suited distribution for EVA of temperature.

APPLICATION

In the present work, a study on EVA of temperature was carried out to estimate extreme (maximum or minimum) temperature for different return periods adopting EV1, EV2, LN2 and LP3 distributions. MoM and MLM are used for determination of parameters of the distributions. Daily temperature data (with missing values) recorded at Hissar for the period 1969 to 2012 was used. The annual maximum and minimum series of absolute, dry bulb and wet bulb was extracted from the daily temperature data and used for EVA.

Table 2 gives the descriptive statistics such as Average ($\bar{x}$), Standard Deviation (SD), Coefficient of Skewness (C_S) and Coefficient of Kurtosis (C_K) of the data series of absolute, dry bulb and wet bulb temperature for Hissar. The statistical tests viz., Wald-Wolfowitz, Mann-Whitney Wilcoxon and Grubbs' are used for checking the randomness, homogeneity and outliers in the annual maximum and minimum series of temperature data. The statistical test results are presented in Table 3.

Analysis Based on Statistical Tests

The statistical tests results of Wald-Wolfowitz and Mann-Whitney Wilcoxon tests indicated that the series of absolute, dry bulb and wet bulb temperature was found to be random and homogeneous respectively. The series was subjected to the Grubb's test which indicated some outliers in the series of annual absolute maximum and wet bulb maximum temperature. However, the entire data was used for modelling considering the importance of the actually observed extremes in the region under consideration.

Data Validation

The scrutiny of the dry bulb and wet bulb temperature data indicated that the data for the years 2002 (January to December) and 2008

(January to December) were missing. Further, the scrutiny of the wet bulb temperature data also showed that the data for the years 1969 (June), 1970 (April), 1971 (March, April and June) and 1972 (May and June) are missing. Similarly, the absolute maximum and minimum temperature for the years 2002 (January to December) and 2008 (May to December) in addition to the absolute maximum temperature for the year 1982 (January and February) and dry bulb minimum temperature for the year 1969 (June) are also found to be missing. Thus, for the series of maximum temperature, the missing data for the month (or) year was imputed by the series maximum value i.e., 48.8 ^{o}C for absolute temperature, 48.0 ^{o}C for dry bulb and 35.4 ^{o}C for wet bulb temperature. Likewise, for the series of minimum temperature, the missing data for the month (or) year was imputed by the series minimum value i.e., -1.5 ^{o}C for absolute temperature, -0.1 ^{o}C for dry bulb and -0.5 ^{o}C for wet bulb temperature. After replacing the missing values by series maximum (or) minimum (AERB, 2008), the entire data series was used for modelling of temperature data.

Table 2. *Descriptive statistics of temperature recorded at Hissar*

Data series	Statistical parameters			
	Average (^{o}C)	SD (^{o}C)	C_S	C_K
Absolute maximum	46.5	1.3	-0.550	0.947
Dry bulb maximum	44.8	1.3	0.118	0.917
Wet bulb maximum	30.4	1.5	1.926	3.383
Absolute minimum	2.6	1.5	-0.439	0.149
Dry bulb minimum	4.0	1.8	-0.824	0.052
Wet bulb minimum	3.0	1.6	-0.436	-0.462

Results and Discussions

The procedures described above for estimating extreme (maximum or minimum) temperature were implemented adopting computer codes. Using the annual maximum (or minimum) series (absolute, dry bulb and wet bulb), the computer code computes the (i) parameters of EV1, EV2, LN2 and LP3 (using MoM and MLM) distributions; (ii) extreme temperature events for different return periods; (iii) GoF tests statistic; and (iv) D-index test values. The estimated absolute, dry bulb and wet bulb maximum temperature (x_T) with Standard Error (SE) using EV1, EV2, LN2 and LP3 distributions for Hissar are presented in Tables 4 to 6 whereas the estimated minimum temperature (x_T) for the series of absolute, dry bulb and wet bulb are given in Tables 7 to 9. From the analysis, it was found that the EV2 distribution is not found to be feasible for fitting the annual minimum series of absolute, dry bulb and wet bulb temperature and therefore the EV2 results are not presented in Tables 7 to 9. The plots of recorded and estimated temperature (maximum or minimum) with confidence limit were presented in Figures 1 to 6.

From Tables 4 to 6, it is noticed that that the estimated absolute, dry bulb and wet bulb maximum temperature using EV2 were consistently higher than the corresponding values of EV1, LN2 and LP3. From Tables 7 to 9, it is noticed that the estimated absolute, dry bulb and wet bulb minimum temperature using EV1 are in lower side when compared to the corresponding values of EV2, LN2 and LP3.

Table 3. *Statistical test results for randomness, homogeneity and outliers*

Data series	Computed value		Theoretical value at 1% significance level	Grubb's test
	Wald-Wolfowitz	Mann-Whitney Wilcoxon		
Absolute maximum	0.839	1.608	2.330	Yes (42.4)
Dry bulb maximum	0.819	2.229	2.330	No outliers
Wet bulb maximum	0.644	0.765	2.330	Yes (25.0)
Absolute minimum	1.144	2.124	2.330	No outliers
Dry bulb minimum	0.642	0.984	2.330	No outliers
Wet bulb minimum	0.642	1.907	2.330	No outliers

Table 4. *Estimated absolute maximum temperature with standard error*

Return Period (Year)	Absolute Maximum Temperature (°C)															
	EV1				EV2				LN2				LP3			
	MoM		MLM		MoM		MLM		MoM		MLM		MoM		MLM	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	46.5	0.2	46.5	0.3	46.5	0.2	46.4	0.2	46.7	0.2	46.7	0.2	46.5	1.2	46.7	1.1
5	47.7	0.3	48.3	0.5	47.7	0.3	48.0	0.3	47.9	0.2	47.9	0.2	47.8	1.3	47.8	1.3
10	48.5	0.5	49.5	0.6	48.5	0.5	49.1	0.4	48.5	0.3	48.5	0.3	48.6	1.5	48.5	1.5
20	49.3	0.6	50.6	0.8	49.4	0.5	50.1	0.6	49.0	0.4	49.0	0.4	49.3	1.7	49.1	1.8
50	50.3	0.7	52.0	1.0	50.5	0.7	51.5	0.7	49.6	0.4	49.6	0.4	50.1	2.0	49.9	2.0
100	51.1	0.8	53.1	1.2	51.3	0.8	52.6	0.8	50.1	0.4	50.0	0.5	50.7	2.2	50.4	2.2
500	52.9	1.1	55.6	1.6	53.3	1.1	55.2	1.1	50.9	0.5	50.9	0.5	52.0	2.6	51.5	2.5
1000	53.6	1.2	56.7	1.7	54.1	1.3	56.3	1.3	51.2	0.6	51.2	0.6	52.6	2.7	52.0	2.7

Table 5. *Estimated dry bulb maximum temperature with standard error*

Return Period (Year)	Dry Bulb Maximum Temperature (°C)															
	EV1				EV2				LN2				LP3			
	MoM		MLM		MoM		MLM		MoM		MLM		MoM		MLM	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	44.9	0.2	44.9	0.3	44.9	0.2	44.7	0.2	45.1	0.2	45.1	0.2	45.1	0.4	45.1	0.6
5	46.3	0.3	46.6	0.5	46.2	0.4	46.1	0.4	46.4	0.3	46.4	0.3	46.4	0.5	46.4	0.7
10	47.2	0.5	47.7	0.6	47.2	0.5	47.1	0.5	47.2	0.3	47.1	0.4	47.2	0.6	47.1	1.0
20	48.1	0.6	48.8	0.8	48.1	0.7	48.0	0.6	47.7	0.4	47.7	0.4	47.7	0.8	47.7	1.1
50	49.2	0.8	50.2	1.0	49.3	0.9	49.2	0.9	48.4	0.5	48.4	0.4	48.4	0.9	48.4	1.3
100	50.0	1.0	51.2	1.2	50.3	1.0	50.2	1.0	48.9	0.5	48.9	0.4	48.9	1.0	48.8	1.5
500	52.0	1.2	53.6	1.5	52.5	1.4	52.5	1.3	49.8	0.6	49.8	0.5	49.8	1.2	49.8	1.7
1000	52.9	1.3	54.7	1.6	53.5	1.5	53.5	1.5	50.2	0.6	50.1	0.6	50.2	1.3	50.1	1.9

Table 6. *Estimated wet bulb maximum temperature with standard error*

Return Period (Year)	Wet Bulb Maximum Temperature (°C)															
	EV1				EV2				LN2				LP3			
	MoM		MLM		MoM		MLM		MoM		MLM		MoM		MLM	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	30.6	0.3	30.6	0.2	30.6	0.2	30.5	0.2	30.9	0.3	30.9	0.3	30.5	0.7	35.9	0.9
5	32.6	0.5	32.2	0.4	32.5	0.3	32.0	0.3	32.8	0.5	32.8	0.3	32.5	1.1	36.8	1.2
10	33.9	0.7	33.2	0.6	33.8	0.5	33.0	0.5	33.9	0.5	33.7	0.5	33.9	1.5	37.4	1.5
20	35.2	0.8	34.2	0.7	35.2	0.6	34.0	0.6	34.8	0.6	34.6	0.5	35.3	1.8	38.0	1.9
50	36.8	1.1	35.5	0.9	37.0	0.8	35.3	0.8	35.8	0.7	35.6	0.6	37.1	2.6	38.8	2.5
100	38.0	1.3	36.5	1.0	38.4	1.0	36.4	0.9	36.5	0.8	36.2	0.7	38.5	2.9	39.4	2.8
500	40.8	1.7	38.7	1.4	41.9	1.4	38.9	1.3	38.0	0.9	37.6	0.9	41.9	4.0	40.9	3.8
1000	42.0	1.9	39.7	1.5	43.5	1.6	40.0	1.6	38.6	1.0	38.2	1.0	43.4	4.5	41.5	4.2

Table 7. *Estimated absolute minimum temperature with standard error*

Return Period (Year)	Absolute Minimum Temperature (°C)											
	EV1				LN2				LP3			
	MoM		MLM		MoM		MLM		MoM		MLM	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	1.1	0.2	2.7	0.3	2.1	0.1	2.6	0.2	2.3	0.2	2.7	0.3
5	-0.4	0.4	1.0	0.4	1.3	0.1	1.6	0.1	1.6	0.2	2.0	0.2
10	-1.4	0.6	-0.1	0.6	1.0	0.1	1.3	0.1	1.4	0.2	1.7	0.2
20	-2.4	0.7	-1.2	0.7	0.8	0.1	1.1	0.2	1.3	0.2	1.6	0.3
50	-3.7	0.8	-2.5	1.0	0.7	0.1	0.9	0.2	1.2	0.2	1.4	0.2
100	-4.6	1.0	-3.5	1.1	0.6	0.1	0.7	0.1	1.2	0.2	1.4	0.3
500	-6.8	1.4	-5.9	1.5	0.4	0.1	0.6	0.1	1.1	0.2	1.3	0.3
1000	-7.7	1.5	-6.9	1.6	0.4	0.1	0.5	0.1	1.1	0.2	1.2	0.2

Analysis Based on GoF Tests

The adequacy of fitting different PDFs for EVA of temperature data was performed by adopting GoF tests using A^2 and KS. The GoF tests results are presented in Table 10 (a and b).

Table 8. *Estimated dry bulb minimum temperature with standard error*

Return Period (Year)	Dry Bulb Minimum Temperature (^{O}C)											
	EV1				LN2				LP3			
	MoM		MLM		MoM		MLM		MoM		MLM	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	2.6	0.3	4.2	0.2	3.6	0.3	4.1	0.3	3.8	0.4	5.2	0.5
5	1.0	0.5	2.6	0.4	2.5	0.2	2.9	0.2	2.9	0.3	4.7	0.4
10	-0.1	0.6	1.6	0.6	2.0	0.1	2.5	0.2	2.7	0.4	4.6	0.5
20	-1.1	0.8	0.5	0.7	1.7	0.1	2.2	0.2	2.5	0.3	4.5	0.6
50	-2.5	0.9	-0.8	0.9	1.5	0.2	1.8	0.1	2.4	0.4	4.4	0.6
100	-3.5	1.1	-1.8	1.0	1.3	0.2	1.7	0.2	2.3	0.4	4.4	0.6
500	-5.8	1.4	-4.0	1.5	1.0	0.1	1.3	0.1	2.2	0.4	4.4	0.8
1000	-6.8	1.6	-5.0	1.6	0.9	0.1	1.2	0.1	2.2	0.4	4.3	0.7

Table 9. *Estimated wet bulb minimum temperature with standard error*

Return Period (Year)	Wet Bulb Minimum Temperature (^{O}C)											
	EV1				LN2				LP3			
	MoM		MLM		MoM		MLM		MoM		MLM	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
2	1.3	0.3	3.1	0.3	2.4	0.2	3.2	0.2	3.0	0.3	3.7	0.4
5	-0.5	0.4	1.3	0.4	1.4	0.1	2.2	0.2	2.2	0.3	3.0	0.4
10	-1.6	0.7	0.2	0.6	1.1	0.1	1.8	0.1	2.0	0.3	2.7	0.3
20	-2.8	0.7	-0.9	0.8	0.9	0.1	1.5	0.1	1.8	0.3	2.5	0.3
50	-4.2	1.0	-2.4	0.9	0.7	0.1	1.3	0.2	1.7	0.3	2.4	0.4
100	-5.3	1.2	-3.4	1.2	0.6	0.1	1.1	0.1	1.6	0.3	2.3	0.4
500	-7.8	1.5	-5.9	1.5	0.4	0.1	0.9	0.2	1.5	0.3	2.1	0.4
1000	-8.9	1.7	-7.0	1.7	0.4	0.1	0.8	0.1	1.5	0.3	2.1	0.4

Table 10(a). *Theoretical and computed values of A^2 statistic using EV1, EV2, LN2 and LP3*

Data Series	Computed Values of A^2 Statistic								Theoretical Value at 1% Level
	EV1		EV2		LN2		LP3		
	MoM	MLM	MoM	MLM	MoM	MLM	MoM	MLM	
Absolute maximum	1.393	1.228	1.717	1.231	0.252	0.348	0.357	0.378	1.038
Dry bulb maximum	0.707	0.756	0.761	1.201	0.653	0.764	0.548	0.653	1.038
Wet bulb maximum	3.730	4.261	3.423	3.983	5.312	5.447	2.895	3.054	1.038
Absolute minimum	0.536	0.192	NF	NF	6.545	7.297	7.832	9.262	1.038
Dry bulb minimum	1.104	0.477	NF	NF	8.724	9.126	7.652	8.777	1.038
Wet bulb minimum	1.489	0.917	NF	NF	7.025	7.769	8.912	9.918	1.038

NF: Distribution is not feasible for fitting to the recorded data.

Table 10(b). *Theoretical and computed values of KS statistic using EV1, EV2, LN2 and LP3*

Data Series	Computed Values of KS Statistic								Theoretical Value at 1% Level
	EV1		EV2		LN2		LP3		
	MoM	MLM	MoM	MLM	MoM	MLM	MoM	MLM	
Absolute maximum	0.092	0.092	0.099	0.101	0.049	0.069	0.066	0.067	0.238
Dry bulb maximum	0.086	0.110	0.085	0.140	0.097	0.102	0.086	0.091	0.238
Wet bulb maximum	0.135	0.146	0.125	0.148	0.233	0.210	0.201	0.232	0.238
Absolute minimum	0.055	0.060	NF	NF	0.285	0.320	0.154	0.178	0.238
Dry bulb minimum	0.095	0.106	NF	NF	0.305	0.342	0.283	0.333	0.238
Wet bulb minimum	0.157	0.138	NF	NF	0.284	0.309	0.175	0.187	0.238

From the GoF tests results, the following observations were drawn from the study:

i) A^2 test confirmed the use of LN2 and LP3 distributions for EVA of absolute and dry bulb maximum temperature for Hissar.

ii) A^2 test also confirmed the use of EV1 (MoM and MLM) and EV2 (MoM) distributions for EVA of dry bulb maximum temperature. A^2 test suggested the EV1, EV2, LN2 and LP3 distributions were not acceptable for EVA of wet bulb maximum temperature.

iii) A^2 test confirmed the applicability of EV1 (MLM) distribution for EVA of absolute, dry bulb and wet bulb minimum temperature.

iv) KS test confirmed the use of EV1, EV2, LN2 and LP3 distributions for EVA of absolute, dry bulb and wet bulb maximum temperature.

v) KS test didn't suggest the applicability of LN2 distribution for EVA of absolute minimum, dry bulb minimum and wet bulb minimum temperature. Thus, the results of GoF show a scatter in clear selection of a particular PDF for EVA of air temperature.

Analysis Based on Diagnostic Test

A diagnostic test of D-index, as described above, was used for the selection of a suitable distribution for EVA of temperature. The D-index values computed for EV1, EV2, LN2 and LP3 distributions by different methods are given in Table 11. From the diagnostic test results, it was noticed that the D-index values of LN2 (MoM) for absolute, dry bulb and wet bulb maximum whereas EV1 (MLM) for the series of absolute, dry bulb and wet bulb minimum temperature were found to be minimum. As the MoM of the probability distributions was noted to be generally less accurate, the selection of best suitable distribution for EVA of temperature was made on the D-index values of probability distributions obtained from MLM.

Table 11. *D-index values of EV1, EV2, LN2 and LP3 distributions*

Data Series	EV1		EV2		LN2		LP3	
	MoM	MLM	MoM	MLM	MoM	MLM	MoM	MLM
Absolute maximum	0.082	0.221	0.089	0.164	0.050	0.050	0.071	0.061
Dry bulb maximum	0.103	0.126	0.107	0.114	0.077	0.078	0.077	0.078
Wet bulb maximum	0.243	0.302	0.264	0.332	0.207	0.225	0.263	0.528
Absolute minimum	3.709	1.177	NF	NF	4.356	4.942	5.485	6.170
Dry bulb minimum	2.234	1.036	NF	NF	2.102	2.744	3.207	6.226
Wet bulb minimum	4.694	2.218	NF	NF	3.170	4.620	5.140	6.710

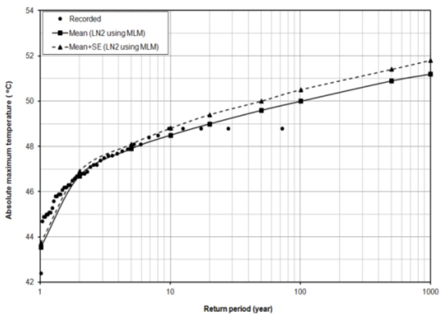

Figure 1. *Plots of estimated Mean and Mean+SE values of absolute maximum temperature using LN2 (MLM) distribution with recorded data*

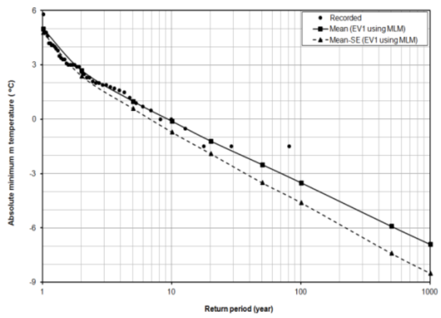

Figure 2. *Plots of estimated Mean and Mean-SE values of absolute minimum temperature using EV1 (MLM) distribution with recorded data*

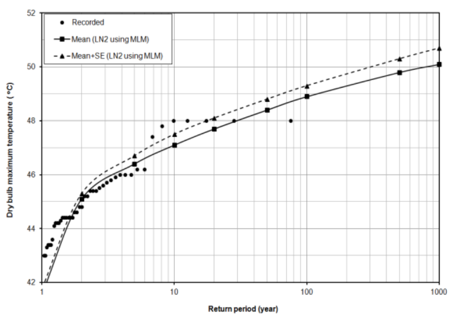

Figure 3. *Plots of estimated Mean and Mean+SE values of dry bulb maximum temperature using LN2(MLM) distribution with recorded data*

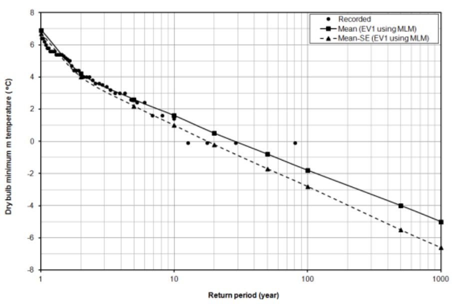

Figure 4. *Plots of estimated Mean and Mean-SE values of dry bulb minimum temperature using EV1 (MLM) distribution with recorded data*

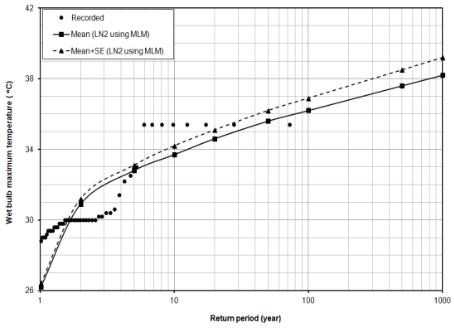

Figure 5. *Plots of estimated Mean and Mean+SE values of wet bulb maximum temperature using LN2 (MLM) distribution with recorded data*

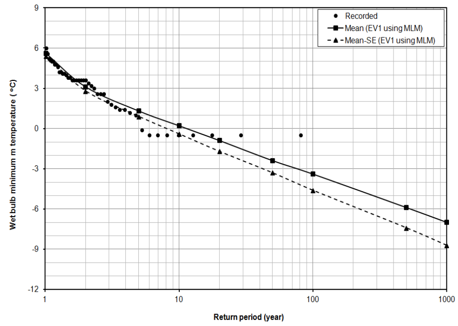

Figure 6. *Plots of estimated Mean and Mean-SE values of wet bulb minimum temperature using EV1 (MLM) distribution with recorded data*

Selection of Probability Distribution

Based on GoF and diagnostic test results, it could be inferred that the LN2 (MLM) could be used for estimation of absolute dry bulb and wet bulb maximum temperature whereas EV1 (MLM) for estimation of absolute, dry bulb and wet bulb minimum temperature. The estimated extreme values for 100-year and 1000-year computed by the respective probability distribution with suitable parameter estimation method are given in Table 12.

Table 12. *Summary of estimated extreme values of temperature for Hissar*

Meteorological Parameter	Probability Distribution	Parameter Estimation Method	Estimated Extreme Values*			
			100-Year		1000-Year	
			Mean	Mean+SE	Mean	Mean+SE
Absolute maximum	LN2	MLM	50.0	50.5	51.2	51.8
Dry bulb maximum	LN2	MLM	48.9	49.3	50.1	50.7
Wet bulb maximum	LN2	MLM	36.2	36.9	38.2	39.2
Absolute minimum*	EV1	MLM	-3.5	-4.6	-6.9	-8.5
Dry bulb minimum*	EV1	MLM	-1.8	-2.8	-5.0	-6.6
Wet bulb minimum*	EV1	MLM	-3.4	-4.6	-7.0	-8.7

**In case of minimum temperature, estimated extreme values may be read as Mean-SE.*

Conclusions

The study on EVA of temperature (absolute, dry bulb and wet bulb) for Hissar adopting EV1, EV2, LN2 and LP3 distributions was carried out with applicable parameter estimation methods. From EVA results of air temperature, the following conclusions were drawn from the study:

i) For the series of absolute, dry bulb and wet bulb maximum temperature, it was found that there is no significant difference between the estimated maximum temperature using LN2 and LP3 distributions.

ii) Suitability of probability distribution was evaluated by GoF (using A^2 and KS) and diagnostic (using D-index) tests. The KS test results confirmed the use of EV1, EV2, LN2 and LP3 distributions for EVA of absolute, dry bulb and wet bulb maximum temperature, Likewise, the A^2 test results suggested the use of EV1 (MLM) distribution for EVA of absolute, dry bulb and wet bulb minimum temperature.

iii) The D-index value of EV1 (MLM) was found to be a minimum when compared to the corresponding values of EV2, LN2 and LP3 distributions for the series of absolute, dry bulb and wet bulb minimum temperature.

iv) On the basis of GoF and diagnostic test results, the study identified the LN2 (MLM) distribution is better suited amongst four distributions studied for estimation of absolute, dry bulb and wet bulb maximum temperature whereas EV1 (MLM) for estimation of absolute, dry bulb and wet bulb minimum temperature for Hissar.

v) Considering the design-life and safety of the hydraulic structure in the vicinity of Hissar region, 1000-year return period Mean+1σ values of absolute, dry bulb and wet bulb maximum temperature of 51.8 °C, 50.7 °C and 39.2 °C respectively; and 1000-year return period Mean-1σ values of absolute, dry bulb and wet bulb minimum temperature of -8.5 °C, -6.6 °C and -8.7 °C respectively is found suitable and suggested for design purposes.

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