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Discrete Optimal Design of Reinforced Concrete Short Columns Using Genetic Algorithms

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Abstract: *This paper aims to optimally design reinforced concrete non-slender columns by using genetic algorithms and explore the effects of the compressive strength of concrete, yield strength of steel, the depth of neutral axis as well as the crossover rate and number of elites of the genetic algorithms on the optimal results. Given conditions are the factored axial load, depth of the neutral axis, compressive strength of concrete and yield strength of steel, length of the column and the size of longitudinal reinforcement and lateral ties. The constraints are built according to the ACI code requirements, by considering the strength requirements of combined axial load and bending, longitudinal reinforcement ratio and clear distance between longitudinal bars. The objective function is to minimize the total cost of longitudinal steel bars, lateral ties and concrete of the column; design variables consist of the column size and number of the tensile reinforcement and compressive reinforcement, all of which are discrete. There are a large number of columns designed in this paper. The numerical results show that different crossover rate or different number of elites always leads to the same optimal results; the higher the compressive strength of concrete is, the lower the minimum cost becomes. However, the yield strength of steel doesn't have the tendency to increase or reduce the optimal cost. In addition, it is also found that the larger the depth of the neutral axis, the smaller the eccentricity becomes and the lower the minimum cost turns out.*

Keywords: *Reinforced Concrete Short Columns, Genetic Algorithms, Optimization*

1. INTRODUCTION

Over the past few decades, many mathematical programming methods have been developed to solve optimization problems, such as calculus-based methods, numerical methods, and random search methods. The calculus-based methods rely mainly on the existence of derivative and smoothness of constraints and objective function, but problems in the real world usually have discontinuities and noisy spaces. The numerical methods can find the optimal value in each space point one at a time, which is very effective when the number of possibilities is very small, but in the big or highly dimensional space it turns very hard to locate the optimal value. Due to the disadvantages of the calculus-based and the numerical methods, the random search methods have gradually become more popular. Random search algorithms are the algorithms involving randomness or probability. They are useful for poorly structured global optimization problems, where the objective function is nonconvex, nondifferentiable, or discontinuous over a continuous, discrete, or mixed continuous-discrete domain. They have been widely applied to a lot of continuous and discrete global optimization problems [1-3].

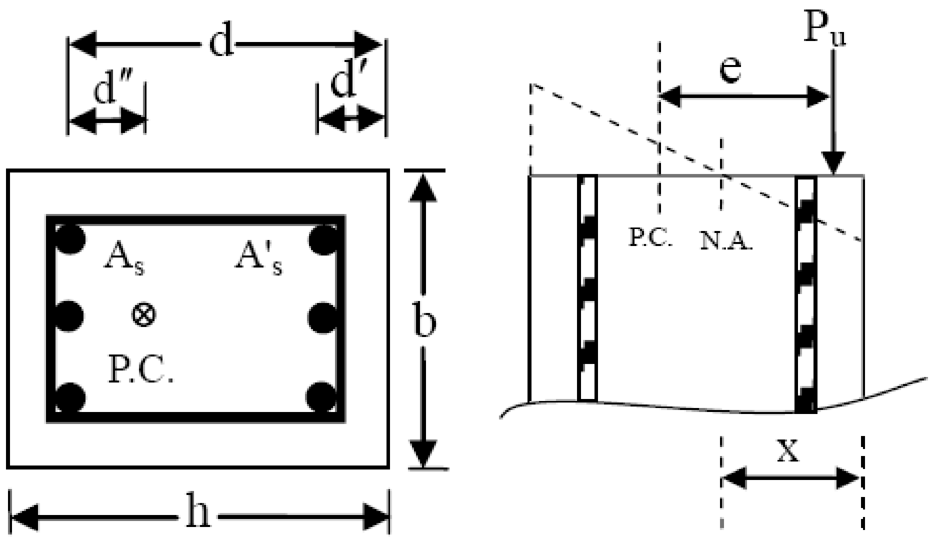
Genetic algorithms, one of the random search methods, are based on Darwin's principles of natural selection [4] and Fisher's genetic theory of the natural selection [5]. They were developed by Holland [6] and described in more detail by Goldberg [7]. Genetic algorithms use random selection to optimize an objective function with respect to variables in the presence of constraints on those variables and have been proven successful for robust searches in complex spaces [8]. There are a number of applications of genetic algorithms in the civil engineering, such as structural reliability analysis [9], optimization of grillages [10], optimization of trusses with a modified genetic algorithm [11], optimization of pile groups by hybrid genetic algorithms [12], optimization of topology and nodal positions of grid shell structures [13] and optimal design of reinforced concrete cantilever beams [14].

2. GENETIC ALGORITHMS

Genetic algorithms play an important role as random search techniques for dealing with complex spaces in many fields. They are one of the best ways to solve the optimization problems whose derivatives are very complicated or difficult to find, and are capable of solving constrained or unconstrained optimization problems, whose constraints can be linear or nonlinear with bounds on the design variables. They use the principles of selection and evolution and work with a set of potential solutions (population) instead of trying to improve a single solution. Natural selection occurs at every life stage of a creature. Over time, creatures change to adapt to the environment to survive and thrive. In many species, adults must compete with each other. The longer they can survive and the more competitive they are, the more offspring they will reproduce. The evolution usually starts from a population of randomly generated individuals. In each generation, the fitness (the objective function value) of every individual (solution) in the population is evaluated. Multiple individuals are stochastically selected from the current population based on their fitness, recombined and randomly mutated to form a new population. A small portion of fittest individuals called elites are kept unchanged and passed on to the next generation. The new population is then used in the next iteration of the algorithm. The algorithm stops when one of the stopping criteria is met, such as the number of generation, the weighted average change in the fitness function value over some generations less than a specified tolerance, no improvement in the best fitness value for an interval of time, etc. The MATLAB Global Optimization Toolbox [15] is employed in this paper to carry out the genetic algorithms. In most engineering problems, the design variables are discrete and genetic algorithms are exactly very good at coping with discrete variables.

3. DESIGN OF REINFORCED CONCRETE COLUMNS

The reinforced concrete columns considered in this paper are non-slender columns whose behavior is controlled by material failure. Given conditions are the factored axial load P_u, neutral axis depth x, compressive strength of concrete f_c' and yield strength of steel f_y, unsupported length of the column and size of longitudinal reinforcement and lateral ties. The constraints are built according to ACI Building Code Requirements for Structural Concrete and Commentary [16], by considering the strength requirements of combined axial load and bending moment, longitudinal reinforcement ratio and the clear distance between longitudinal bars. The objective function is to minimize the total cost of longitudinal steel bars, lateral ties and concrete of the column with clear height l_n; discrete design variables are the width b and thickness h of the column as well as the number of the tensile reinforcement N_1 and the number of compressive reinforcement N_2. The column geometry is shown in Fig. 1, where e is the eccentricity, d the effective depth, N.A. neutral axis, P.C. the plastic centroid and A_s and A_s' are the areas of tension and compression reinforcement, respectively. The units of force and length in the following formulas are kgf (=9.81N) and cm, respectively. The constraints required for genetic algorithms are listed as follows.



(a) (b)

Fig1. *Column geometry: (a) plane; (b) elevation*

3.1. Combined Compression and Bending

Let β_1 be concrete stress block depth factor and E_s the modulus of elasticity of steel. The nominal axial load strength is defined as

$$P_n = C_c + C_s - T_s \tag{1}$$

where $C_c = 0.85 f'_c \beta_1 x b$, $C_s = A'_s(f'_s - 0.85 f'_c)$ and $T_s = A_s f_s$.The stress f'_s in the compression reinforcement can be expressed as

$$f'_s = E_s \frac{(x - d')0.003}{x} \le f_y \tag{2}$$

and the stress f_s in the tension reinforcement is given by

$$f_s = E_s \frac{(d - x)0.003}{x} \le f_y \tag{3}$$

The moment about the tension reinforcement becomes

$$P_n(d'' + e) = 0.85 f'_c \beta_1 x b(d - \frac{\beta_1 x}{2}) + C_s(d - d') \tag{4}$$

The eccentricity *e* is not independent and can be found by substituting Eqs. (1) to (3) into Eq. (4). In practice, eccentricities are unavoidable due to slight inaccuracies in the layout of columns, unsymmetrical loading of slabs in adjacent spans, imperfections in the alignment, etc. Hence, the ACI code specifies a reduction of 20% in the axial load for tied columns, i.e., the maximum nominal axial load capacity

$$P_{n,\max} = 0.8[0.85 f'_c A_g + (A_s + A'_s)(f_y - 0.85 f'_c)] \tag{5}$$

where $A_g = b \times h$ is the gross area of the concrete section. Hence, the constraints for the combined axial load and moment are written as

$$P_n \leq P_{n,\max} \tag{6}$$

$$P_u \leq \phi P_n \tag{7}$$

and $$P_u\, e = M_u \leq \phi M_n = \phi\,(P_n)e \tag{8}$$

where P_u is the factored axial load, M_u is the maximum factored moment the column can be subjected to and the strength reduction factor

$$\phi = 0.65 + 0.25 \times \frac{\varepsilon_s - \varepsilon_y}{0.005 - \varepsilon_y} \leq 0.9 \tag{9}$$

provided that ε_s and ε_y are tensile strain and yield strain in the tension reinforcement, respectively. In fact, if Eq. (7) is satisfied, Eq. (8) will be automatically satisfied.

3.2. Longitudinal Reinforcement Ratio

Most columns are subjected to both axial load and moment. To ensure some ductility, a minimum of 1% reinforcement must be provided and to avoid reinforcement congestion the reinforcement should not be more than 8%; therefor, the total amount of reinforcement in the column has to satisfy the following condition

$$0.01 \leq \frac{A_s + A_s'}{A_g} \leq 0.08 \tag{10}$$

where $A_s = N_1 \times A_b$ and $N_2 \times A_b$ provided that A_b is the cross-sectional area of reinforcement.

3.3. Clear Distance between Longitudinal Bars

To prevent honeycombing and ensure that the wet concrete mix passes through the steel bars without separation, the requirement for the clear distance S_1 between longitudinal bars is given by

$$S_1 \geq \max\ (1.5 d_b,\ 3.81)\ cm \tag{11}$$

where d_b is the diameter of the longitudinal bar.

3.4. The Slenderness Effect

For unbraced columns, the slenderness effect can be neglected if the slenderness ratio

$$\frac{k l_u}{r} \leq 22 \tag{12}$$

where k is the effective length factor and is taken as 1 in this paper, l_u is the unsupported length of the column, r is the radius of gyration.

3.5. The Spacing of the Ties

Tied columns have closed lateral ties spaced uniformly across the column. The spacing of the ties must be close enough to prevent spalling of the concrete cover or local buckling of the longitudinal bars and far enough not to interfere with the setting of the concrete. The ACI code specifies that vertical spacing of ties shall not exceed 16 times the longitudinal bar diameter, 48 times the tie bar diameter or least lateral dimension of the column; therefore, the spacing of the ties is equal to

$$S_2 = \min\ (16 d_b,\ 48 d_{bt},\ b)\ \text{cm} \tag{13}$$

where d_{bt} is the diameter of the tie. After the spacing of the ties is known, the total number of ties along the unsupported length of the column can then be calculated.

4. NUMERICAL RESULTS

This paper uses genetic algorithms to optimally design reinforced concrete non-slender columns, where the unsupported length of the column is assumed to be 350 cm, the concrete cover is 4 cm and No. 8 longitudinal bars and No. 4 ties are used. Given conditions are factored axial load P_u, neutral axis depth x, compressive strength of concrete f_c' and yield strength of steel f_y. Discrete design variables are the width b and thickness h of the column as well as the number N_1 of the tensile reinforcement and the number N_2 of compressive reinforcement. Based on the often-used materials and customs in Taiwan, this paper selects three kinds of yield strength f_y of the longitudinal reinforcement: 2800 kgf/cm^2 (40 ksi), 3500 kgf/cm^2 (50 ksi) and 4200 kgf/cm^2 (60 ksi) whose price is 13200 NT$/ton as well as four kinds of compressive strength f'_c of the concrete: 210 kgf/cm^2 (3000 psi), 280 kgf/cm^2 (4000 psi), 350 kgf/cm^2 (5000 psi) and 420 kgf/cm^2 (6000 psi) whose prices are 1950 NT$/m^3, 2150 NT$/m^3, 2350 NT$/m^3 and 2450 NT$/m^3, respectively. There are seven kinds of neutral axis depth x: 10 cm, 15 cm, 20 cm, 25 cm, 30 cm, 35 cm and 40 cm; five kinds of factored load: 100ton, 150ton, 200ton, 250ton and 300ton. Based on the reasonable combinations of the above-mentioned values, there are totally 290 cases selected to be designed in this paper.

This paper employs the MATLAB global optimization toolbox to carry out genetic algorithms. The constraints and objective function for the algorithms are built according to the discussion in Section 3. Given the factored axial load P_u=150 ton, neutral axis depth x=15 cm, compressive strength of concrete f_c'=280 kgf/cm^2 and yield strength of steel f_y=4200 kgf/cm^2, this paper first lets the crossover rate range from 0.75 to 0.9 and the number of elites from 2 to 10 in a bid to explore their effects on the optimal results. It turns out that different crossover rates and elites always lead to the same results: b=43 cm, h=54cm, N_1=2, N_2 = 5 and the cost=3199 NT$. To run genetic algorithms of the MATLAB software, some parameters need to be specified beforehand, here are the values selected: the population size =100, crossover rate = 0.8 and elite number = 10. Furthermore, all the individuals (solutions) are encoded as integers; “Rank” is used as the scaling function that scales the fitness values based on the rank of each individual; “Roulette” is the selection function to choose parents for the next generation; “Two-point crossover” is used as the crossover method to form a new child for the next generation; The “Adaptive Feasible Function” is chosen as the mutation function to make small random changes in the individuals and ensure that linear constraints and bounds are satisfied. In order to reach the global minimum, genetic algorithms are executed 20 times for each case. Among the results of 20 executions, the minimum cost is taken as the optimal result for this case. For example, Table 1 shows the 20 results for the case of f_c'= 280 kgf/cm^2, f_y= 2800 kgf/cm^2, P_u= 200 ton and x= 25 cm, for which the minimum cost is NT$ 3272 and b=49 cm, h=54 cm, N_1=2 and N_2=4. To explore the effects of the compressive strength of concrete on the optimal results, f_c' is varied and the other three given conditions are fixed. Some examples are shown in Table 2, which can be plotted in Fig. 2. The results show that the higher the compressive strength of concrete, the lower the minimum cost turns out. To explore the effects of the yield strength of steel on the optimal results, f_y is varied and the other three given conditions are fixed. Some examples are shown in Table 3, which can be plotted in Fig. 3. The results show that the yield strength of steel doesn't have the tendency to increase or reduce the minimal cost. To explore the effects of the depth of the neutral axis on the optimal effects, x is varied and the other three conditions are fixed. Some examples are shown in Table 4, which can be plotted in Fig. 4. The results show that the larger the depth of neutral axis x, the smaller the eccentricity becomes and the lower the minimum cost turns out. From the 290 case designed in this paper, it is found that the number of tension reinforcement always approaches two, as shown in Tables 1-4, which is the lower bound set for longitudinal reinforcement when the genetic algorithms are executed.

Table1. *The 20 results for the case of* f_c' = *280 kgf/cm*^{2}, f_y= *2800 kgf/cm*^{2}, P_u = *200 ton and* x = *25 cm.*

Cycle	f_c' (kgf/cm^2)	f_y (kgf/cm^2)	P_u (ton)	x (cm)	e (cm)	b (cm)	h (cm)	N_1	N_2	Cost (NT$)
1	280	2800	200	25	22.2	47	57	2	4	3299
2	280	2800	200	25	22.2	47	57	2	4	3299
3	280	2800	200	25	22.2	47	57	2	4	3299
4	280	2800	200	25	20.3	49	54	2	4	3272
5	280	2800	200	25	22.2	47	57	2	4	3299
6	280	2800	200	25	20.3	49	54	2	4	3272
7	280	2800	200	25	20.3	49	54	2	4	3272
8	280	2800	200	25	20.3	49	54	2	4	3272
9	280	2800	200	25	20.3	49	54	2	4	3272
10	280	2800	200	25	22.2	47	57	2	4	3299
11	280	2800	200	25	21.5	48	56	2	4	3306
12	280	2800	200	25	22.2	47	57	2	4	3299
13	280	2800	200	25	20.3	49	54	2	4	3272
14	280	2800	200	25	20.3	49	54	2	4	3272
15	280	2800	200	25	22.2	47	57	2	4	3299
16	280	2800	200	25	20.3	49	54	2	4	3272
17	280	2800	200	25	20.3	49	54	2	4	3272
18	280	2800	200	25	20.3	49	54	2	4	3272
19	280	2800	200	25	22.8	47	58	2	4	3336
20	280	2800	200	25	20.3	49	54	2	4	3272

Table2. *Some examples of fixing* f_y, P_u *and* x *and varying* f_c' .

e. g.	f_c' (kgf/cm^2)	f_y (kgf/cm^2)	P_u (ton)	x (cm)	e (cm)	b (cm)	h (cm)	N_1	N_2	Cost (NT$)
1	210	4200	100	10	36.7	56	56	2	7	3992
	280	4200	100	10	36.7	53	54	2	5	3627
	350	4200	100	10	37.0	46	54	2	5	3502
	420	4200	100	10	37.5	41	54	2	5	3346
2	210	4200	150	15	28.3	49	54	2	6	3455
	280	4200	150	15	29.1	43	54	2	5	3199
	350	4200	150	15	30.3	41	54	2	4	3085
	420	4200	150	15	31.2	41	54	2	3	2979
3	210	4200	200	20	28.3	48	60	2	6	3625
	280	4200	200	20	28.3	42	59	2	5	3325
	350	4200	200	20	29.3	40	59	2	4	3214
	420	4200	200	20	27.1	42	55	2	3	3065
4	210	4200	250	25	28.5	53	67	2	7	4292
	280	4200	250	25	27.0	46	64	2	6	3879
	350	4200	250	25	22.8	52	56	2	4	3687
	420	4200	250	25	22.0	48	54	2	4	3501
5	210	4200	300	30	35.5	54	83	2	7	4963
	280	4200	300	30	23.5	57	64	2	6	4432
	350	4200	300	30	17.5	54	54	2	6	4058
	420	4200	300	30	18.6	53	54	2	4	3744

Table3. *Some examples of fixing f_c', P_u and x and varying f_y.*

e.g.	f_c' (kgf/cm^2)	f_y (kgf/cm^2)	P_u (ton)	x (cm)	e (cm)	b (cm)	h (cm)	N_1	N_2	Cost (NT$)
1	210	2800	100	10	30.7	54	54	2	6	3649
	210	3500	100	10	36.5	58	58	2	6	3972
	210	4200	100	10	36.7	56	56	2	7	3992
2	210	2800	150	15	25.7	51	54	2	6	3532
	210	3500	150	15	26.8	46	54	2	6	3338
	210	4200	150	15	28.3	49	54	2	6	3455
3	210	2800	200	20	24.9	58	58	2	6	3972
	210	3500	200	20	22.5	53	54	2	7	3794
	210	4200	200	20	28.3	48	60	2	6	3625
4	210	2800	250	25	25.0	63	64	2	7	4635
	210	3500	250	25	25.1	60	63	2	7	4454
	210	4200	250	25	28.5	53	67	2	7	4292
5	210	2800	300	30	25.4	67	70	2	8	5289
	210	3500	300	30	24.2	66	67	2	8	5098
	210	4200	300	30	35.5	54	83	2	7	4963

Table4. *Some examples of fixing f_c', f_y and P_u and varying x.*

e. g.	f_c' (kgf/cm^2)	f_y (kgf/cm^2)	P_u (ton)	x (cm)	e (cm)	b (cm)	h (cm)	N_1	N_2	Cost (NT$)
1	210	4200	150	10	43.4	74	74	2	10	6216
	210	4200	150	15	28.3	49	54	2	6	3455
	210	4200	150	20	27.9	40	56	2	5	2979
	210	4200	150	25	24.1	40	55	2	5	2949
	210	4200	150	30	20.2	41	54	2	4	2775
2	350	2800	200	10	36.66	71	71	2	9	6429
	350	2800	200	15	24.93	49	54	2	6	3825
	350	2800	200	20	24.01	45	55	2	4	3310
	350	2800	200	25	21.07	44	55	2	3	3041
	350	2800	200	30	20.63	39	57	2	3	2912

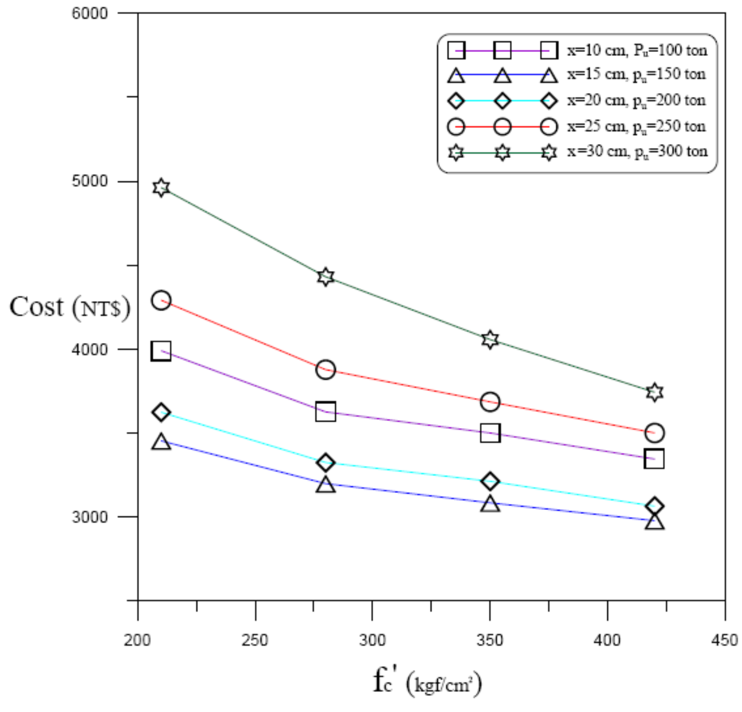


Figure2. *The minimum cost with f_y = 4200 kgf/cm^2 fixed and f_c' varied*

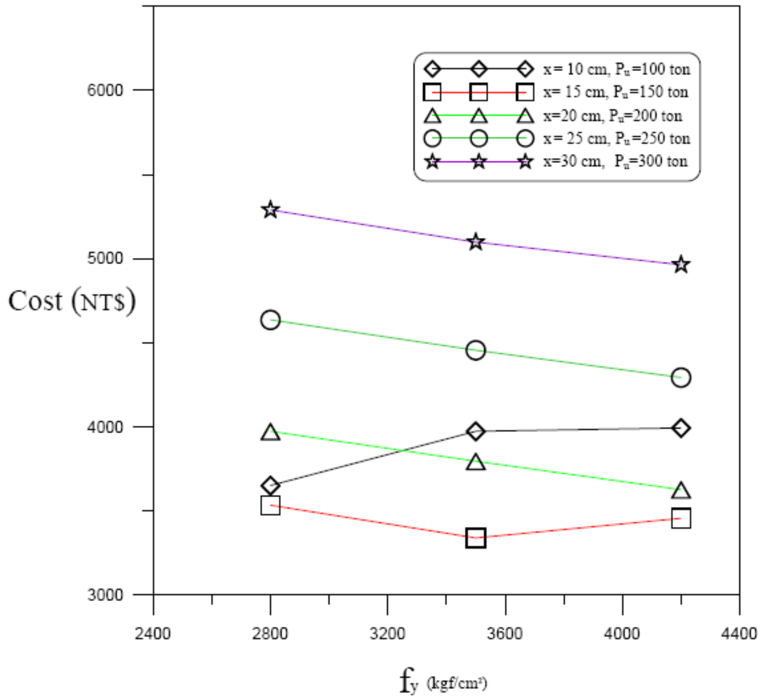


Figure3. *The minimum cost with f_c' =210 kgf/cm^2 fixed and f_y varied*

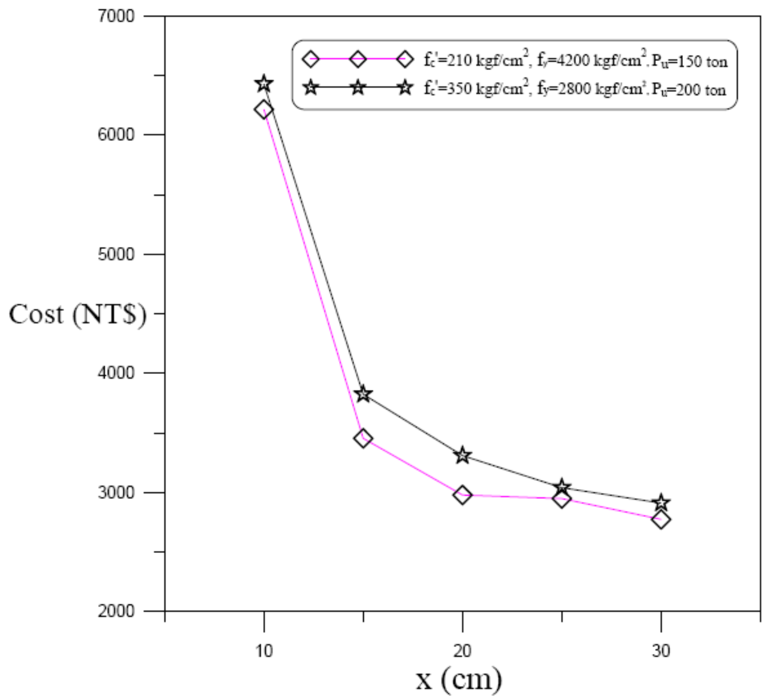


Figure4. *The minimum cost with x varied*

5. Conclusions

Traditionally, columns are designed by trial-and-adjustment method, which is time-consuming. The method used in this paper not only can design reinforced concrete non-slender columns efficiently but also obtain the columns with the minimum cost. From the numerical results, the principal conclusions may be summarized as follows:

(1) Within the scope of giving conditions in this paper, the compressive strength of concrete has influence on the minimum cost: the higher the compressive strength of the concrete, the lower the minimum cost, while the yield strength of steel doesn't have direct influence on the cost.

(2) In order to have the minimum cost, the number of tension reinforcement always approaches the lower limit that is set to be two.

(3) The larger the depth of neutral axis *x* is, the smaller the eccentricity becomes and the lower the minimum cost turns out.

(4) Once the factored axial load, neutral axis depth, compressive strength of concrete and yield strength of steel are given, the genetic algorithms can quickly and optimally design the reinforced concrete non-slender columns without going through the tedious column design procedure.

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