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A General Solution for the Conjectures Related To the Infinity of Prime Numbers that Take any Special Form

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Abstract: *Further analysis of the demonstration in Porras-Ferreira and Andrade (2014) that prime numbers always occupy one of eight columns in a 30-column array and infinite rows such that $P_n = [1,7,11,13,17,19,23,29] + 30n$, for $n \geq 0$ (except for 1 which is not prime for $n = 0$ and the primes 2, 3 and 5, the only primes that occupy columns 2, 3 and 5) showed that composite numbers N_c generated in the eight columns where prime numbers orderly locate, follow a defined pattern originated in the same primes located in the different columns. This pattern has two characteristics; the first one establishes the row number n where a composite numbers will be located and the second characteristic gives the value of the prime factors of each composite number.*

The method above permits the prediction of the location for cells where composite numbers will occur and their prime factors in a simple way. In a simple illustration, prime number 7, being the smallest of the order equation of prime numbers (eight columns in a 30-column array), establishes a six-row sequence where primes can be located and one row where 7 will always be a prime factor to infinity accompanying with other prime factors that originate composite numbers. Each prime factor, different from 7, originates a similar sequence, but being greater, their sequence can intersect the sequence of 7 in some n, leaving always free rows in the eight columns where prime numbers will locate to infinity in the eight columns in a 30-column array.

Dirichlet´s theorem establishes that there are infinite primes of the form $an + b$, for $n = 1, 2, \ldots$, where a, b are primitive integer numbers. This equation is the same of the order of prime numbers $p_n = [1,7,11,13,17,19,23,29] + 30n$, where $a = 30$, $b = [1,7,11,13,17,19,23,29]$ and n is the same for both equations, therefore if the mathematical transformation of any conjecture to the form $P_n = [1,7,11,13,17,19,23,29] + 30n$, for $n \geq 0$, is possible, such form will contain infinite primes and the conjecture will be true according to Dirichlet's theorem. According to that, the demonstration of the infinitude of Mersenne primes, Fermat primes and $4x + 1$ primes are shown as well the demonstration that there are not odd perfect numbers.

Keywords: *Dirichlet's Theorem, Mersenne Primes, Fermat primes, $4x + 1$ primes, Perfect Numbers, Prime Numbers Formation.*

1. Introduction

Many of the questions around prime numbers that remained open until recently have been solved by means of using the found order of prime numbers as expressed in Porras-Ferreira and Andrade (2014) [1]. Often having an elementary formulation, many of these conjectures have withstood a proof for decades: for all four of Landau's problems from 1912 [2] solutions were offered in [3]. Goldbach's conjecture: "every even integer greater than 2 can be expressed as the sum of two primes" [4] has an explicit solution in [3] including the "weak" Goldback's conjecture. The twin prime conjecture: "there are infinitely many pairs of primes whose difference is 2" [5] and that there are infinite prime numbers of the form $a^2 + 1$" (e.g. Euler, 1760) [6] were solved using the found order as well as the 1379 Conjecture Porras-Ferreira and Andrade-Amaya, (2015) [3]. Some other famous patterns of primes have also been conjectured with like the order the French monk Marin Mersenne devised of the form $2^p - 1$, with p a prime [7] from which today the Great Internet Mersenne Prime Search (GIMPS) is looking for another even larger prime [8][9], the conjecture whether there are infinite Fermat's primes of the form $F_y = 2^{2^y} + 1$ [10], and many other proposed patterns that are not mentioned here for simplicity.

In Porras-Ferreira and Andrade (2015) [1] it was established that prime numbers in a systematic form always occupy one of eight columns in a 30-column array and infinite rows such that $P_n =$

$[1,7,11,13,17,19,23,29] + 30n$, for $n \geq 0$ (except for 1 which is not prime for $n = 0$ and the primes 2, 3 and 5, the only primes that occupy the columns 2, 3 and 5). Later, in Porras-Ferreira and Andrade-Amaya (2015) [3] this order array was tested and used for solving several conjectures related to the sequences for prime numbers. The natural inquietude was to follow the study of the behavior of the location for composite numbers in the eight columns where prime numbers are positioned in the 30-column array and the application of Dirichlet's theorem [11] in the eight columns in a 30-column array where the primes are formed, to solve other conjectures related to prime numbers as infinitude of Mersenne primes [12], Fermat primes [13] and primes $4x + 1$ for $x \geq 1$ [14].

In antiquity, several mathematicians thought that numbers of the form $2^p - 1$ were primes for all p primes [15]. Hudalricus Regius in 1536 [15], demonstrated that $2^{11} - 1$ was not a prime. Pietro Cataldi in 1603 [15], correctly verified that $2^{17} - 1$ and $2^{19} - 1$, were both primes, but also enunciated that for $p = [23, 29, 31, 37]$. Fermat in 1640 [16] demonstrated that Cataldi were wrong for $p = [23, 27]$. Later, Marin Mersenne, a French monk, in 1644 [17], correctly established that for $p = [2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257]$ the resultant numbers were primes but mistakenly established that for the rest primes $p > 257$ the resultant numbers were composite numbers. Pervouchine in 1873 [16], verified that for $p = 61$ the resultant number were prime and Power in 1900 [6] probed for $p = [89, 107]$. Finally in 1947, a complete verification was done, resulting in the correct list $p = [2, 3, 5, 7, 13, 17, 19, 31, 61, 67, 89, 107, 257]$.

Because of his work, all prime numbers of the form $2^p - 1$, where p is a prime number, receive the name of Mersenne primes (M_p) [18], denoted as:

$$M_p = 2^p - 1$$

Mersenne primes are related to the even perfect numbers. A perfect number is a positive integer that is equal to the sum of its proper positive divisors, that is, the sum of its positive divisors excluding the number itself (also known as its aliquot sum). Equivalently, a perfect number is a number that is half the sum of all of its positive divisors (including itself) i.e. $\sigma_1(n) = 2n$.

This definition is ancient, appearing as early as Euclid's Elements (VII.22)[1] where it is called "*τέλειος ἀριθμός"* (perfect, ideal, or complete number). Euclid also proved a formation rule (IX.36) whereby $p(p + 1)/2$ is an even perfect number whenever p is what is now called a Mersenne prime. Much later, Euler proved that all even perfect numbers are of this form. It is not known whether there are any odd perfect numbers [19], or if infinitely many even perfect numbers exist.

Fermat conjectured in 1650 that every Fermat number is Fermat prime. The much more commonly encountered Fermat numbers are a special case [20],[21] and [22], given by the binomial number of the form $F_y = 2^{2^y} + 1$ and Eisenstein in 1844 proposed as a problem the proof that there are an infinite number of Fermat primes (Ribenboim 1996, p. 88 [23]). At present, however, the only Fermat numbers F_y for $y \geq 5$ for which primality or compositeness has been established are all composite. Fermat believed that all numbers of the above form are prime numbers; that is, that F_y is prime for all integral values of y. This is indeed the case for $y = [0.1.2.3.4]$. However, the Swiss mathematician Leonhard Euler (1707–83) showed that Fermat's conjecture is false for $y = 5$: $2^{2^5} = 2^{32} + 1 = 4{,}294{,}967{,}297$, which is divisible by 641. Using computers, mathematicians have not yet found any Fermat primes for y greater than 4. So far, Fermat's original hypothesis seems to have been wrong. The search continues for Fermat numbers F_y that are prime for $y \geq 4$.

2. The Pattern Produced by the Prime Number 7 in The Position of Composite Numbers in the Prime Numbers Order Array

The prime number 7 establishes an initial pattern in the Prime Numbers Order Array, where composite numbers will locate themselves every seven rows in each of the eight columns where prime numbers are. The method to find the initial cell n_1, where 7 as prime factor will appear and

[1] Euclid's Elements (Ancient Greek: Στοιχεῖα *Stoicheia*) is a mathematical and geometric treatise consisting of 13 books written by the ancient Greek mathematician Euclid in Alexandria c. 300 BC

furthermore, the form of calculating the n-row where composite numbers that have 7 as a prime factor was established in Porras-Ferreira and Andrade [1] in equations (1) and (2).

Taken into account that equation (1), form prime numbers P_n and composite numbers N_c:

$$\begin{bmatrix} P_n = [1,7,11,13,17,19,23,29] + 30n \; for \; n \geq 0 \\ N_c = [1,7,11,13,17,19,23,29] + 30n \text{ for } n \geq 1 \end{bmatrix} \tag{1}$$

There will be no prime numbers in cells:

$$n = \begin{bmatrix} n_1 + p_1 k \\ n_1 + p_2 k \\ n_1 + \cdots k \\ n_1 + p_n k \end{bmatrix} \Rightarrow k \geq 1 \tag{2}$$

Where n are the rows where there are no primes, n_1 is the first row of any of the eight columns where primes are located and where p_n appears for the first time. That is to say that from cell n_1, the multiplication table of prime numbers $p_1, p_2, \dots p_n$ that conform the composite number N_c is created.

The n row from where no primes will be generated can be predicted by using the residue until reaching 0. The example for following procedure for prime $p = 7$ column 1 is:

In row 1 column 1: $31 \div 7 \equiv 3 \; (mod \; 7)$. In row 2 column 1: $61 \div 7 \equiv 5 \; (mod \; 7)$. That is to say the increment factor is $5 - 3 = 2$. The residue 5 become 0 in row 3 column 1 (5+2=7): $91 \div 7 \equiv 0 \; (mod \; 7)$, in other words in row$n_1 = 3$ of column 1, there will be a multiple of 7 and in $n = 3 + 7k$ for $k \geq 0$ there will be no prime numbers in column 1 (only composite numbers where one of its prime factor is 7). Same procedure can be followed to establish in what row n_1 and n of the other seven columns composite numbers with 7 as one of its prime factors will be located. Table I shows cells n_1 of equation (2) where 7 appears for the first time as prime factor of the composite numbers in each of the eight columns in Equation (1). The other primes that compose the composite number in row n_1, will begin a different sequence of rows n where there will be composite numbers. Table I also shows the values for cells n for two of the initial prime factors that compose the composite number N_c and the respective n rows what are not primes for $k \geq 0$.

Table I. *Rows n_1 and n of each column where composite numbers with 7 as prime factor will be located in the array according to equations (1) and (2) and values of N_c for $k \geq 0$.*

Rows n	Column 1	Column 7	Column 11	Column 13	Column 17	Column 19	Column 23	Column 29
n_1	3	7	5	4	2	1	6	3
N_c in n_1 for 7	7*13	7*31	7*23	7*19	7*11	7*7	7*29	7*17
$n = n_1 + 7k$ for $p_1 = 7$	$3 + 7k$	$7 + 7k$	$5 + 7k$	$4 + 7k$	$2 + 7k$	$1 + 7k$	$6 + 7k$	$3 + 7k$
n for other primes (Eq. 2)	$3 + 13k$	$7 + 31k$	$5 + 23k$	$4 + 19k$	$2 + 11k$	$1 + 7k$	$6 + 29k$	$3 + 17k$

In Figure 1 for the case of prime number 7 it is possible to see the repeating pattern in each column where prime numbers are formed.

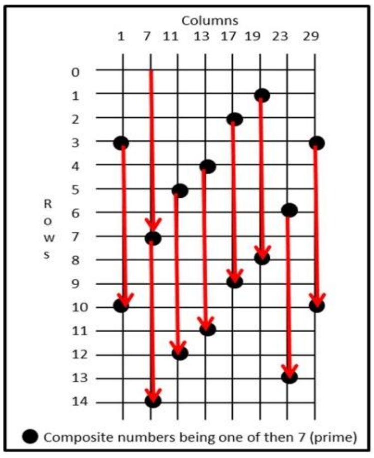

Fig1. *Repeating pattern locations of composite numbers being one of them the number 7 (prime)*

Table II shows the case for the composite numbers that have the prime 11 as one of its factors; similar to Table 1 for the prime 7 and so on it can be shown how the other primes that will be appearing for the first time as factor of N_c will follow the same behavior.

TableII. *Rows n_1 and n of each column where composite numbers with 11 as prime factor will be located in the array according to equations (1) and (2) and values of N_c for $k \geq 0$.*

Rows n	Column 1	Column 7	Column 11	Column 13	Column 17	Column 19	Column 23	Column 29
n_1	4	6	11	8	2	21	4	6
N_c in n_1 for 11	11*11	11*17	11*31	11*23	11*7	11*59	11*13	11*19
$n = n_1 + 11k$ for $p_1 = 11$	$4 + 11k$	$6 + 11k$	$11 + 11k$	$8 + 11k$	$2 + 11k$	$21 + 11k$	$4 + 11k$	$6 + 11k$
n for other primes (Eq. 2)	$4 + 11k$	$6 + 17k$	$11 + 31k$	$8 + 23k$	$2 + 7k$	$21 + 59k$	$4 + 13k$	$6 + 19k$

Taking into account that Equation (1) for composite numbers N_c , can be expressed as:

$$N_c = p_m([1,7,11,13,17,19,23,29] + 30k) \text{ for } k \geq 0 \quad (3)$$

Where p_m corresponds to the value of the new prime numbers that would be appearing in n_1 of equation (2) and that haven't appeared before as a factor of N_c. Table III gives some examples for column 1: the prime number 31 appeared for the first time in row $n_1 = 1$ and will appear again in $n = n_1 + 31k = 1 + 31 = 32$ for$k = 1$, as factor of the composite number $N_c = p_m(1 + 30k) = 31(1 + 30) = 31 * 31$ for$k = 1$. The same way it is fullfilled for all values of n_1 and p_m that will be resulting from equations (2) and (3).

TableIII. *Examples for equations (2) and (3) for* $k = [1,2,3]$

k	p_m	n_1	n		N_c	
1				32		31*31
2	31	1	1+31k	63	31(1+30k)	31*61
3				94		31*91
1				124		61*61
2	61	63	63+61k	185	61(31+30k)	61*91
3				246		61*121
1				276		91*91
2	91	185	185+91k	367	91(61+30k)	91*121
3				458		91*151

In the same way, it is possible to calculate in what cell in the other columns will appear the prime number as a factor of composite numbers using the Residue System. For example the prime numbers p_1 in column 1 and p_7 in column 7, will appear as a factor in the others columns according to table IV.

TableIV. *Examples for prime numbers p_n in column 1, will appear as a factor in the others columns*

Concept	Column 1	Column 7	Column 11	Column 13	Column 17	Column 19	Column 23	Column 29
Form	30*n* +1	30*n* +7	30*n* +11	30*n*+13	30*n*+17	30*n*+19	30*n*+23	30*n*+29
Value	p_1	$7p_1$	$11p_1$	$13p_1$	$17p_1$	$19p_1$	$23p_1$	$29p_1$
Cell	*n*	7*n*	11*n*	13*n*	17*n*	19*n*	23*n*	29n
Compisite		210*n*+7	330*n*+11	390*n*+13	510*n*+17	570*n*+19	690*n*+23	87*n*+29
Value	$13p_7$	p_7	$23p_7$	$19p_7$	$11p_7$	$7p_7$	$29p_7$	$17p_7$
Cell	*n*+3	*n*	*n*+5	*n*+4	*n*+2	*n*+1	*n*+6	*n*+3
Composite	30(*n*+3)+1		30(*n*+5)+11	30(*n*+4)+13	30(*n*+2)+17	30(*n*+1)+19	30(*n*+6)+23	30(*n*+3)+29

3. Proof of the Existence of a Unique Pattern in the Prime Numbers Given by Equation (1)

The importance of equations (1) and (2), in the case for number 7, which is the smallest prime in the equation, establishes a repetitive pattern every seven rows, leaving six rows where prime numbers will be located in the eight columns and one row where definitively never prime numbers will be located in the 30-column array, as shown in Table 1. This pattern repeats to infinity. Prime numbers greater than 7 that would appear as factors of the composite number N_c also originates repetitive patterns that will locate cells n_1 and n where composite numbers form as shown in Tables 1 and 2, complying equations (2) and (3). These patterns will have a greater separation between one cell to other cells n where prime numbers will be located with respect to the pattern of the prime number 7 because they are primes greater than 7. Eventually these patterns would be super imposed among them when they take the same value of cell n in equation (2).

The above confirms that there is only one pattern defined in equation (1) for all prime numbers, always existing cells where prime numbers will be located in each of the eight columns of the array to infinity and warranty that in one of them there will be prime numbers located. Table V shows some examples up to row $n = 59$ where equations (1) (2) and (3) are verified, for prime numbers Equation (1), as for composite numbers Equations (2) and (3).

TableV. *The location and formation of prime and composite numbers in the 30-column array according to equations (1), (2) and (3).*

Rows *n*	Column 1	Column 7	Column 11	Column 13	Column 17	Column 19	Column 23	Column 29
0		7	11	13	17	19	23	29
1	31	37	41	43	47	7*7	53	59
2	61	67	71	73	7*11	79	83	89
3	7*13	97	101	103	107	109	113	7*17
4	11*11	127	131	7*19	137	139	11*13	149
5	151	157	7*23	163	167	13*13	173	179
6	181	11*17	191	193	197	199	7*29	11*19
7	211	7*31	13*17	223	227	229	233	239
8	241	13*19	251	11*23	257	7*37	263	269
9	271	277	281	283	7*41	17*17	293	13*23
10	7*43	307	311	313	317		17*19	7*47
11	331	337	11*31	7*49*7*7	347	349	353	359
12	19*19	367	7*53	373	13*29	379	383	389
13	17*23	397	401	13*31	11*37	409	7*59	419
14	421	7*61	431	433	19*23	439	443	449
15	11*41	457	461	463	467	7*67	11*43	479
16	13*37	487	491	17*29	7*71	499	503	509
17	7*73	11*47	521	523	17*31	23*23	13*41	7*7*11
18	541	547	19*29	7*79	557	13*43	563	569
19	571	577	7*83	11*53	587	19*31	593	599
20	601	607	13*47	613	617	619	7*89	17*37
21	631	7*7*13	641	643	647	11*59	653	659
22	661	23*29	11*71	673	677	7*97	683	13*53
23	691	17*41	701	19*37	7*101	709	23*31	719
24	7*103	727	17*43	733	11*67	739	743	7*107
25	751	757	761	7*109	13*59	769	773	19*41
26	11*71	787	7*113	13*61	797	17*47	11*73	809
27	811	19*43	821	823	827	829	7*119(7*17)	839
28	29*29	7*11*11	23*37	853	857	859	863	11*79
29	13*67	877	881	883	887	7*127	19*47	29*31
30	17*53	907	911	11*83	7*131	919	13*71	929
31	7*7*19	937	941	23*41	947	13*73	953	7*137
32	31*31	967	971	7*139	977	11*89	983	23*43
33	991	997	7*11*13	17*59	19*53	1009	1013	1019
34	1021	13*79	1031	1033	17*61	1039	7*149	1049
35	1051	7*151	1061	1063	11*97	1069	29*37	13*83
36	23*47	1087	1091	1093	1097	7*157	1103	1109
37	11*101	1117	19*59	1123	7*7*23	1129	11*103	17*67
38	7*163	31*37	1151	1153	13*89	19*61	1163	7*167
39	1171	11*107	1181	7*13*13	1187	29*81	1193	11*109
40	1201	17*71	7*173	1213	1217	23*53	1223	1229
41	1231	1237	17*73	11*113	29*43	1249	7*179	1259
42	13*97	7*181	31*41	19*67	1277	1279	1283	1289

43	1291	1297	1301	1303	1307	7*11*17	13*101	1319
44	1321	1327	11*11*11	31*43	7*191	13*103	17*79	17*71
45	7*193	23*59	1361	29*47	1367	37*37	1373	7*197
46	1381	19*73	13*107	7*11*19	11*127	1399	23*61	1409
47	17*83	13*109	7*7*29	1423	1427	1429	1433	1439
48	11*131	1447	1451	1453	31*47	1459	7*11*19	13*113
49	1471	7*211	1481	1483	1487	1489	1493	1499
50	19*79	11*137	1511	17*89	37*41	7*7*31	1523	11*139
51	1531	29*53	23*67	1543	7*13*17	1549	1553	1559
52	7*223	1567	1571	11*11*13	19*83	1579	1583	7*227
53	37*43	1597	1601	7*239	1607	1609	1613	1619
54	1621	1627	7*233	23*71	1637	11*149	31*51	17*97
55	13*127	1657	11*151	1663	1667	1669	7*239	23*73
56	41*41	7*241	19*89	1693	1697	1699	13*131	1709
57	29*59	19*103	1721	1723	11*157	7*13*19	1733	37*47
58	1741	1747	17*103	1753	7*251	1759	41*43	29*61
59	7*253	1777	13*137	1783	1787	1789	11*163	7*257

Table V has the following information:

- Contains nine columns, the first is n and the other are the eight columns of the Array where prime numbers are located and where composite numbers also appear.
- Colors represent a different prime factor, for example Green represents the composite numbers that have the prime 7 as factor, Grey shows the prime 11 etc.
- It can be observed how these composite numbers follow the behavior established in equation (3) as they form in Table 4. In some cases cell with one pattern coincide with another pattern for composite numbers. For example cell 31 of column 1 coincide with the pattern generated by the prime 7 as factor of the composite number that formed in row 3 and the prime number 19, factor of the composed number that formed in row 12.
- Prime Numbers are located in the cells in White color.
- Another important aspect is that each column where the factor 7 generates for the first time, the other factor of the composite number is precisely the prime number in the other columns, for example, for column 1: number 7 as a factor is accompanying by the 13, the first prime in column 13: for column 7: number 7 is accompanying by the 31, which is the first prime in column 1; for column 11: number 7 is accompanying by number 23 which is the first prime in column 23 and so on, until column 29, where number 7 is accompanying by number 17, which is the first prime of column 17. Same accompanying complies for the rest of the prime numbers that appear for the first time, as composite number, following equations (2) and (3) as it was shown also for prime 11 in Table 2.

4. Relation of the Primes Order Array and Dirichlet's Theorem

Dirichlet's theorem [11] demonstrated that: *If a and b are primitive integer numbers among them, then, the arithmetic progression $an + b$ for $n = 1, 2, \ldots$, contain infinite primes.*

Taking from Equation (1) $a = 30$, $b = [1,7,11,13,17,19,23,29]$, $n = 1, 2, \ldots$, according to Dirichlet's theorem, the infinity of prime numbers in each of the eight columns of the array is then confirmed. In that sense, any conjecture for prime numbers that can be expressed in the form of Equation (1), by the rule of Dirichlet's theorem it must have infinite primes and it is true. In this sense, the solutions of conjectures about the infinity Mersenne primes $2^p - 1$, Fermat's primes $2^{2^y} + 1$ for $y \geq 0$ and primes $4x + 1$ for $x \geq 1$will be demonstrated.

3.1. The implication of Dirichlet´s theorem and prime order array to proof there are infinity Mersenne Primes [24]

Table VI ([24] and [25]); show the 48 Mersenne primes, found in the Great Internet Mersenne Prime Search Program (GIMPS), the last discovered in 2013. The Great Internet Mersenne Prime Search (GIMPS) is a collaborative project of volunteers[2] who use freely available software to search

[2] Volunteer computing is an arrangement in which people (**volunteers**) provide computing resources to projects, which use the resources to do distributed computing and/or storage.

for Mersenne prime numbers. GIMPS is said to be one of the first large scale distributed computing projects over the Internet for research purposes.

TableVI. *48 Mersenne Primes found until 2013, [24] [25]*

No.	p	Digits in p	M_p	Digits in M_p	Discoverer	By
1	2	1	3	1	B C	Unknown
2	3	1	7	1	B C	Unknown
3	5	1	31	2	B C	Unknown
4	7	1	127	3	B C	Unknown
5	13	2	8191	4	1456	Unknown
6	17	2	131071	6	1588	Cataldi
7	19	2	524287	6	1588	Cataldi
8	31	2	2147483647	10	1772	Euler
9	61	2	2305843009213693951	19	1883	Pervushin
10	89	2	618970019…449562111	27	1911	Powers
11	107	3	162259276…010288127	33	1914	Powers
12	127	3	170141183…884105727	39	1876	Lucas
13	521	3	686479766…115057151	157	30/01/1952	Robinson
14	609	3	531137992…031728127	183	30/01/1952	Robinson
15	1.279	4	104079321…168729087	386	25/06/1952	Robinson
16	2.203	4	147597991…697771007	664	07/10/1952	Robinson
17	2.281	4	446087557…132836351	687	09/10/1952	Robinson
18	3.217	4	259117086…909315071	969	08/09/1957	Riesel
19	4.253	4	190797007…350484991	1.281	03/11/1961	Hurwitz
20	4.423	4	285542542…608580607	1.332	03/11/1961	Hurwitz
21	9.689	4	478220278…225754111	2.917	11/05/1963	Gillies
22	9.941	4	346088282…789463551	2.993	16/05/1963	Gillies
23	11.213	5	281411201…696392191	3.376	02/06/1963	Gillies
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35	1.398.269	7	814717564…451315711	420.921	13/11/1996	GIMPS / J. Armengaud
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48	57.885.161	8	581887266…724285951	17.425.170	25/01/2013	GIMPS / Curtis Cooper

An analysis of Table VI shows that:

1. All Mersenne primes end in 1 or 7. Porras-Ferreira and Andrade in [1] established the way on how prime numbers form through Equation (1).

Then, it could be thought that Mersenne Primes, only would have the form $M_n = 30n + [1, 11, 7, 17]$. However, further detail discard the forms $30n + [11, 17]$, letting only the forms $30n + [1, 7]$ for Mersenne primes, because:

- Mersenne Primes cannot end in 11.

Proof:

$2^p - 1 = 30n + 11$

$2^p - 12 = 30n$

3 divide-12, but it won't divide 2^p which is an even integer no multiple of 3. Therefore $M_n \neq 30n + 11$.

- They can´t end in 17 either.

Proof:

$2^p - 1 = 30x + 17$

$2^p - 18 = 30x$

3 divide-18, but it doesn´t divide 2^p which is an even integer no multiple of 3. Therefore $M_n \neq 30n + 17$.

- Mersenne Primes can finish in 1.

Proof:

$2^p - 1 = 30x + 1$

$2(2^{p-1} - 1) = 30n$

$2^{p-1} - 1 = 15n$

$16^{\frac{p-1}{4}} - 1 = 15n$

Making $a = \frac{p-1}{4}$, being a integer and $p \equiv 1 \pmod 4$, would be:

$16^a - 1 = 15n$ where $16^a - 1 \equiv 0 \pmod{15}$. (16 elevated to any power will always end in 6, that subtracting 1 would end in 5, likewise it is divisible by 3, being the sum of its digits multiples of 3. Examples, for $a = 1$, then $16 - 1 = 15$, for $a = 2$ then $256 - 1 = 255$ multiple of 3 and 5.

For this case $n = \frac{16^a - 1}{15}$ and $p \equiv 1 \pmod 4$.

- Mersenne primes can end in 7.

Proof:

$2^p - 1 = 30n + 7$

$8\left(2^{\frac{p-3}{4}} - 1\right) = 30n$

$4(2^{\frac{p-3}{4}} - 1) = 15n$

Making $a = \frac{p-3}{4}$, being a integer and $p \equiv 3 \pmod 4$ would be:

$4(16^a - 1) = 15n$ where $16^a - 1 \equiv 0 \pmod{15}$. For this case $n = \frac{4(16^a - 1)}{15}$ and $p \equiv 3 \pmod 4$.

2. Prime numbers p can have the form:

$$p = 30n + [1, 7, 11, 13, 17, 19, 23, 29]$$

With the restrictions expressed in 1c. and 1d. therefore the forms of p would be:

$$p = \begin{Bmatrix} 30n + [1, 13, 17, 29] \text{ for } n \text{ even} \\ 30n + [7, 11, 19, 23] \text{ for } n \text{ odd} \end{Bmatrix} \equiv 1 \pmod 4$$

$$p = \begin{Bmatrix} 30n + [7, 11, 19, 23] \text{ for } n \text{ even} \\ 30n + [1,13,17,29] \text{ for } n \text{ odd} \end{Bmatrix} \equiv 3\ (mod\ 4)$$

3. Mersene primes are infinite, according to Dirichlet's theorem [11] and the form of prime numbers given by Porras-Ferreira and Andrade (2014) [1], where:

$M_n = 2^p - 1 = 30n + 1$ for $n = \frac{16^a - 1}{15}$, $a = \frac{p-1}{4}$ and $p \equiv 1$ (mod 4), or

$M_n = 2^p - 1 = 30n + 7$ for $n = \frac{4(16^a - 1)}{15}$, $a = \frac{p-3}{4}$ and $p \equiv 3$ (mod 4)

with exception of 3 and 7 that are Mersenne Primes also.

3.2. The Implication of Dirichlet´s Theorem and Prime Order Array to Proof There are Infinite Primes 4x + 1 For x ≥ 1

On the conjecture of the existence of infinite primes of the form $4x + 1$ for $x \geq 1$ a demonstration is not known, as there is a way of demonstrating that there are infinite primes $4x - 1$ for $x \geq 1$. [14].

Demonstration that there are infinite primes $4x + 1$ for $x \geq 1$:

From Equation (1) for $n \geq 1$:

- $1 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for even n, where $x = 30n/4$
- $7 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for odd n, where $x = (6 + 30n)/4$
- $11 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for odd n, where $x = (10 + 30n)/4$
- $13 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for even n, where $x = (12 + 30n)/4$
- $17 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for even n, where $x = (16 + 30n)/4$
- $19 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for odd n, where $x = (18 + 30n)/4$
- $23 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for odd n, where $x = (28 + 30n)/4$
- $29 + 30n \equiv 1\ (mod\ 4) = 4x + 1$ for even n, where $x = (28 + 30n)/4$

According to Dirichlet's theorem [11] and the form of prime numbers given by Porras-Ferreira and Andrade (2014) [1], there are infinite solutions of prime numbers ending in [1, 13, 17, 29] for even n and for prime numbers ending in [7, 11, 19, 23] for odd n, therefore there are infinite primes $4x + 1$ for $x \geq 1$.

Because it has been able to effect the transformation $4x + 1$ to the general form of the prime numbers given by Equation (1), leads to the conclusion that there are infinite prime of the $4x + 1$ form. Table VII shows some examples of primes for different values of odd and even n (Equation (1)), calculating the respective x value for $4x + 1$ primes. The respective prime number meets the two forms simultaneously (Equation (1) and $4x + 1$ forms):

Table VII. *Examples of primes $4x + 1$ compared with primes of Equation (1), for values of odd and even n*

Form	n	Primes	x	Form
	71	2137	534	
$7 + 30n$	101	3037	759	$4x + 1$
	361	10837	2709	
	71	2141	535	
$11 + 30n$	101	3041	760	$4x + 1$
	107	3221	805	
	3	109	27	
$19 + 30n$	101	3049	762	$4x + 1$
	105	3169	792	
	71	2153	538	
$23 + 30n$	113	3413	853	$4x + 1$
	361	10853	2713	
	40	1201	300	
$1 + 30n$	112	3361	840	$4x + 1$
	148	4441	1110	
	40	1213	303	
$13 + 30n$	112	3373	843	$4x + 1$

	386	11593	2898	
	40	1217	304	
$17 + 30n$	120	3617	904	$4x + 1$
	386	11597	2899	
	40	1229	307	
$29 + 30n$	130	3929	982	$4x + 1$
	228	6869	1717	

Q.E.D.

Also let p be an odd prime. Then $x^2 \equiv -1 \; mod \; p$ has a solution (i.e. -1 is a quadratic residue of p) iff $p \equiv 1 \; mod \; 4$. In [3] was demonstrated infinite primes exist that form $p = x^2 + 1$, then infinite primes exist that form $4x + 1$. By another form, suppose there are finitely many $p_1, p_2, \dots, p_k$. If $N = (2p_1p_2 \dots p_k)^2 + 1$ where $N > 1$ and odd, therefore exist and odd prime $p|N$, where $p = p_m$ for some $1 \leq m \leq k$, but $(2p_1p_2 \dots p_k)^2 \equiv -1 \; mod \; p$ where $p \equiv 1 \; mod \; 4$ then $p|(2p_1p_2 \dots p_k)^2$ and $p|1$ is false. So there exist infinite p where $p > p_k$ and it is possible to continue in the same way as $p_k \to \infty$. This procedure is similar to the demonstration that exist infinite primes made by Euclid and special cases of a remarkable theorem due to Dirichlet [11] and prime order array [1].

The other way to prove this is by reduction *ad absurdum*, i.e. assuming that there is a prime p congruent $1 \; mod \; 4$, which is the largest. As a result, if $p_1, \dots, p_n$ are primes congruent $1 \; mod \; 4$, then $p_i \leq p$ for all $i = 1, \dots, n$. On the other hand, $p = 1 + 4x$ for some $x \in \mathbb{N}$. Then, taking into account the fundamental theorem of arithmetic, x can be represented as

$$x = {p_1}^{k_1} \times {p_2}^{k_2} \times \dots \times {p_r}^{k_2}$$

Where $k_1, k_2, \dots, k_r$, are non-negative integers, $r \leq n$.

Defining a number q as well:

$$q = 1 + 4xp$$

$$= 1 + 4{p_1}^{k_1} \times {p_2}^{k_2} \times \dots \times {p_r}^{k_2} \times p$$

Here, it is obvious that, q is not divisible by any prime, since it would always result residue 1, then q is divisible only by 1 and by itself, i.e., q is prime, which turns out to be contradictory since we had assumed that p was the largest prime, and we have found that q is prime, $q > p$ and $q \equiv 1 \; mod \; 4$, so there are infinite primes $1 + 4x$.

3.3. The Implication of Dirichlet´S Theorem and Prime Order Array to Proof there are Infinite Fermat Primes $F_y = 2^{2^y} + 1$ For $y \geq 1$

For this conjecture on whether there are infinite Fermat primes $F_y = 2^{2^y} + 1$, the solution is derived from the previous demonstration, since the factor 2^{2^y} can be transform to $4x$:

$2^{2^y} = 4^z$, where $z = 2^{y-1}$, $4x = 4^z$ and $x = 4^{z-1}$.

Given that z is always even, 4^z will always end in 6, therefore Fermat primes form is: $30n + 17$. As there are infinite prime solutions $30n + 17 \equiv 1 \; (modulo \; 4) = 4x + 1$, for even n, therefore must exist infinite solutions of prime $4^z + 1 \equiv 2^{2^y} + 1 \equiv 4x + 1 \equiv 30n + 17$ for $x = 4^{z-1} = 4^{2^{y-1}-1}$, $z = 2^{y-1}$ and $n = \frac{4^z - 16}{30} = \frac{4^{2^{y-1}} - 16}{30}$, for $y \geq 2$. Note for $y = [0, 1]$, Fermat primes are 3 and 5 and they are the only different primes from form $30n + 17$

Table VIII gives examples of Fermat primes, where the form $2^{2^y} + 1$, $4x + 1$ and $30n + 17$ are met.

Table VIII. *Examples of Fermat primes* $F_y = 2^{2^y} + 1$

y	z	4^z	n	x	$4^z + 1 \equiv 2^{2^y} + 1 \equiv 4x + 1 \equiv 30n + 1$
2	2	16	0	4	17
3	4	256	8	64	257
4	8	65536	2184	16384	65567

To date the largest Fermat prime number known is 65537. Verifications for $z = [16, 32, 64, 128, 256, 512, 1024]$ have been made and the resulting numbers are composites, however as primes of form $30n + 17$ are infinite there is no reason for values of $n = \frac{4^z - 16}{30}$ with higher values of $z > 1024$, can be Fermat primes where n is an exponential progression and in Equation (2), n is an arithmetic progression.

Q.E.D.

Whether there is heuristic argument that suggests there is only a finite number of them. This argument is to due to Hardy and Wright [26].

Recall that the Prime Number Theorem says $\Pi(x) \sim \frac{x}{\log x}$, where $\Pi(x)$ is the number of primes $\leq x$. Hence $\Pi(x) < \frac{Ax}{\log x}$ for some constant A, and the probability that x is a prime is at most $\frac{A}{\log x}$. For $2^{2^y} + 1$, the probability that it is a prime is $\leq \frac{A}{\log(2^{2^y}+1)} \leq \frac{A}{\log 2^{2^y}} = \frac{A}{2^y \log x} \leq \frac{A}{2^y}$. Hence, the expected number of primes in this form is $\leq \sum_0^\infty \frac{A}{2^y} = 2A$ which is a finite number.

However, it is necessary to be careful that there arguments do not prove that there are really only finitely many Fermat primes. After all, they are only heuristic, as it can be seen in a similar arguments:

1. Use the same reasoning to argue that there are infinitely many twin primes. Recall the Prime Number Theorem can be stated using limit: $\lim_{x\to\infty} \frac{\Pi(x)}{\frac{x}{\log x}} = 1$. Hence give $\varepsilon > 0$, there exists a number X such that $1 - \varepsilon < \frac{\Pi(x)}{\frac{x}{\log x}}$ for all $x > X$

Thus, the probability that x and $x + 2$ are both primes is $\frac{\Pi(x)}{x} \cdot \frac{\Pi(x+2)}{x+2} > \frac{1}{\log x} \cdot \frac{1}{\log(x+2)} (1-\varepsilon)^2$ for $x > X$. So the expected number of twin primes is $> \sum_0^m \frac{1}{x}(1-\varepsilon)^2 + \sum_m^\infty \frac{1}{x}(1-\varepsilon)^2$ which diverges. There are infinitely many primes in the form of $2^x + 1$.

Using the exact same argument, the expected number of primes in this form is $> \sum_0^m \frac{1}{x}(1-\varepsilon)^2 +$ $m\infty 1x(1-\varepsilon)2$ which diverges.

But $2^x + 1$ primes and $2^{2^y} + 1$ primes are the same set. This latter argument suggests Hardy and Wright's argument does not take into account of the properties of Fermat primes. It is to say that the variable x is not that random. It works largely because gaps between successive Fermat numbers are extremely large. Nevertheless, given any number (even a number of a particular form), it is more likely to be a composite than prime. Therefore, bounding the probability of it being a prime by a lower bound gives a weaker argument that bounding it from above then there are infinitely many Fermat prime as it was demonstrated.

3.4. Analysis on Perfect Numbers

A positive integer n is called a perfect number if it is equal to the sum of all its positive integers divisors excluding n [27]. Mersenne primes are connected with perfect numbers thru the equation

$$2^{p-1}(2^p - 1) = 2^{p-1} M_p = n \tag{4}$$

This demonstration was made by Euclid 2500 years ago showing that $2^p - 1$ should be a Mersenne Prime. In the XVIII century Euler probe the convers meaning that every perfect number has to have the form of Equation (4). That demonstration is in [14], [28] and [29].

The reason why perfect numbers comply with the above for M_n and the other primes don´t can be demonstrated as follows:

Proof:

- Let n be a perfect number and M_p a prime number M different from a Mersenne prime.
- Equation (4) would be: $2^{p-1}M = n$

- The sum of the factors of 2^{p-1} is $2^{p-1}-1$, (for example the sum of the factors of $2^{5-1}=16$ is $2+4+8+1=15=2^{5-1}-1$.
- The sum of all divisors of n for p is $(2^{p-1}-1)$ and the sum of its factors by M would be $(2^{p-1}-1)M$.
- Therefore the sum of all divisors of n would be: $(2^{p-1}-1)M+(2^{p-1}-1)+2^{p-1}=2^{p-1}M-M+2(2^{p-1})-1=n=2^{p-1}M$
- $M=2(2^{p-1})-1=2^p-1$ ………………………………………..……...according to 4.
- $M=M_p$ Therefore what was assumed in 1 is false ……………….…….…..according to 6.

Q.E.D.

The above demonstration left all even numbers n in equation (4) as perfect and as it was said, Euler proves that all even perfect numbers have that form.

The second question analyzed is to prove that odd perfect numbers cannot exist. For example Roberts, T. 2008 [30] has done studies on the form of an odd perfect number; Goto, T; Ohno, Y. 2008 [31] established that odd perfect numbers have a prime factor exceeding 10^8 and Ochem, Pascal and Rao, Michaël., 2012 [32] established that odd perfect numbers are greater than 10^{1500}, but it is not necessary to go there according to the following demonstration:

Proof:

- Let $n=n_p$ an odd perfect number.
- The fundamental theorem of integer numbers say that all integer number can be decomposed in its prime factors. As n is odd its prime factors are odd, $n=p_1p_2p_3 \dots p_m$ where p_m is the $m-esim$ odd prime factor that compose n.
- Initially assuming that $m=2$, meaning there are 2 prime factors where $p_2 \geq p_1$ and $n=p_1p_2$
- By definition of a perfect number, for $n=n_{p_2}$ be a perfect number (n_{p_2} is a perfect number with two prime factors), must be equal to the sum of all its integer positive divisors excluding n, therefore:

$$n_{p_2}=1+p_1+p_2$$

- According to 1, 3 and 4

$$n=p_1p_2=n_{p_2}=1+p_1+p_2$$

but,

$$n=p_1p_2=(p_1-1)p_2+p_2=n_{p_2}=1+p_1+p_2$$

therefore:

$$(p_1-1)p_2=1+p_1$$

Being $p_1 \geq 3$ and $p_2 \geq p_1 \geq 3$, then $(p_1-1)p_2 \neq 1+p_1$ and $n=(p_1-1)p_2+p_2 \neq n_{p_2}=1+p_1+p_2$, therefore, there are not odd perfect numbers with two odd prime factors.

- Assuming that $m>2$ where $p_1 \leq p_2 \leq p_3 \leq \cdots p_m$ and $n=p_1p_2p_3 \dots p_m$ that can be reduced to:

  $n/p_a=p_1p_2$ where $p_a=p_3 \dots p_m$
- According to 5. $\frac{n}{p_a} \neq n_{p_2}$, therefore $n \neq n_{p_2}p_a=n_{p_m}=1+p_1+p_2+p_3+\dots p_m+$ *p2p3… pm+p1p3… pm+p1p2… pm+…+p1p2p3… pm−1.* In conclusion, there cannot be an odd number equal to a perfect number.

Q.E.D.

The complete exercise for $m=3$ is as follow,

$$n = p_1 p_2 p_3$$

$$n_{p_3} = 1 + p_1 + p_2 p_3 + p_2 + p_1 p_3 + p_3 + p_1 p_2$$

$$\begin{aligned} n &= p_2 p_3 + (p_1 - 1) p_2 p_3 = p_2 p_3 + (p_1 - 1) p_3 + (p_1 - 1)(p_2 - 1) p_3 \\ &= p_2 p_3 + (p_1 - 1) p_3 + (p_1 - 1)(p_2 - 1) + (p_1 - 1)(p_2 - 1)(p_3 - 1) \\ &= 1 - p_1 - p_2 - p_3 + p_1 p_2 + p_1 p_3 + p_2 p_3 + (p_1 - 1)(p_2 - 1)(p_3 - 1) \end{aligned}$$

$$n = n_{p_3}$$

$$(p_1 - 1)(p_2 - 1)(p_3 - 1) = 2p_1 + 2p_2 + 2p_3$$

but:

$$(p_1 - 1)(p_2 - 1)(p_3 - 1) \neq 2(p_1 + p_2 + p_3) \tag{5}$$

therefore:

$$n \neq n_{p_3}$$

Note: In Equation (5) it is easy to prove that when $p_{1,2} = 3$ and $p_3 \leq 7$ then:

$(p_1 - 1)(p_2 - 1)(p_3 - 1) < 2(p_1 + p_2 + p_3)$ and when $p_1 \geq 3$ and $p_{2,3} \geq 5$ then:

$$(p_1 - 1)(p_2 - 1)(p_3 - 1) > 2(p_1 + p_2 + p_3)$$

The effect of multiplication in the left side is greater than the effect of sum in the right side of Equation (5).

5. Conclusion

Prime numbers are infinite along each one of the eight columns where they are located in the 30-column array. This means that composite numbers that may form in Equation (1) never will fill all cells of each of those columns, complying the patterns given by Equations (2) and (3) to infinity. It can happen, for example in some cells within the repetitive pattern of the factor 7, all cells can be filled for all composite numbers, but there will be others going to infinity where this won't happen. Therefore there will always be prime numbers occupying the cells where there are no composite numbers as $n \to \infty$. Because of the above, the pattern of order of the prime numbers given by Equation (1) is confirmed.

Applying Dirichlet's theorem the infinity of the prime numbers along each of the columns in Equation (1) is confirmed, so that if any conjecture on prime numbers obey to a pattern that can be algebraically transformed to the form of Equation (1), there will be warranty of the certainty of such conjecture, since there is no limit where the conjecture and the equivalent p values are the same as n row →∞.

It was demonstrated that Mersenne primes only end in 1 and 7 therefore they are of the form $30n + [1, 7]$ for $n \geq 1$, with exception of 3 and 7 that are Mersenne primes. Using Dirichlet theorem and what was established in Porras-Ferreira and Andrade [1], it was demonstrated that Mersenne primes are infinite. Also it was demonstrated that primes $4x + 1$ and Fermat primes are infinite as special cases of a remarkable theorem due to Dirichlet [11] and prime order array [1].

Likewise, Mersenne primes are part of all even perfect numbers, as was demonstrated by Euclid and Euler, it was demonstrated that primes different of Mersenne primes can't be part of even perfect numbers. Finally it was demonstrated that odd perfect numbers cannot exist.

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