## Analytical Decoupling Control Strategy Using a Unity Feedback Control Structure for MIMO Processes with the Delay

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**Abstract:** In this project, a two-level of-adaptability (TDOF) control arrangement for multi-data/multi-yield (MIMO) systems with different time deferrals is proposed. Introductory, a novel inversing decoupler design procedure in perspective of the ideal decoupling is proposed for decoupling the MIMO structure with distinctive time delays into a couple of same independent single-data/single yield (SISO) structures. By then the H2 degree essential subordinate (PID) controller in light of the inside model control (IMC) is associated with the got SISO systems. Finally, the second controller is made by decoupled close circle trade limit. Furthermore, in light of the way that the TDOF control structure can disengage the unsettling impact from the reference, the controller of the reference circle can be used to upgrade the reference response unreservedly. Entertainments are given to speak to the ampleness of the proposed setup approach.

### **1. INTRODUCTION**

Most mechanical procedures are fundamentally multivariable systems. Compelling control of multivariable procedures is a troublesome issue in the control range. Due to circle associations, those settled control systems for SISO systems can't be specifically reached out to MIMO systems. Generally, control designers take care of these issues utilizing single-circle PID controllers overlooking the impacts of the cooperation's. These multi-circle control methodologies have advanced through years of experience in light of the fact that they are straightforward and execute. They are sufficient when the cooperation's in distinctive channels of the procedure are unassuming. In any case, when collaborations are huge, the coupling impacts ought not be precluded. For this situation, it is fitting to pick the concentrated control plan. One compelling methodology of unified control is a decoupling structure joined with an inclining decentralized controller. The subsequent shut circle exchange capacity system is decoupled. Through the years, decoupling control has been broadly explored. There are two sorts of plan for decoupling control. One is static decoupling, the other is element decoupling. Static decoupling certifications complete decoupling just for low frequencies, which may not be adequate for a decent execution. An identical exchange capacity lattice was presented for decoupler configuration. Garrido et al. proposed an augmentation of the reversed and improved decoupling methodology, which took into account more adaptability in picking the exchange capacity of the decoupled obvious procedure. Chen et al. gave another control strategy for MIMO first request time defer non-square systems in view of the decoupling technique. The decoupler was planned taking into account the Mooer-Penrose Pseudo-converse of the enduring state pick up grid of the procedure model. B. T. Jevtovic et. Al proposed a configuration calculation of same controller in all circles for two-info two-yield (TITO) system in light of perfect decoupler . Zhang et al introduced a diagnostic answer for the ideal decoupling control issue. No weighting capacities need to be picked; the acquired controller is given in diagnostic structure. This work concentrate on a standout amongst the most expanded types of traditional decoupling called perfect decoupling. This methodology got significant consideration in both control hypothesis and modern practice for a very long while. Then again, it concentrates on TITO systems. The creators have discovered not very many distributed works in which perfect decoupling is connected to procedures that are bigger than a  $2\square$  2 systems. Additionally, we can alter the tune parameter in the composed controller to avoid unreasonable control endeavors. Once the H2 ideal controller has been intended for the decoupled SISO system, a TDOF controller is created to abstain from surpassing overshoot. The TDOF control structure can separate the aggravation from the reference; the controller of the

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reference circle can be utilized to enhance the reference reaction freely. This project is sorted out as takes after: Section 2 gives the proposed same controller in all circles taking into account perfect decoupling. In Section 3, the TDOF controller for same controller in all circles is produced. Reproduction results are given to represent the proposed technique in Section 4. At last, a few conclusions are given in Section

#### 2. SAME CONTROLLER IN ALL LOOPS BASED ON IDEAL DECOUPLING

#### **2.1 Design of the Decoupler**

Assume that the process is a linear stable non-singular system with n inputs and n outputs. The schematic of the decentralized control system with a decoupling matrix is shown in Fig. 1, where D(s) is the decoupler and C(s) is the feedback controller.



Fig. 1. Block diagram of a decoupling control system

The process transfer function matrix G(s) and decoupler D(s) are defined as follows.

$$G(s) = \begin{bmatrix} g_{11}(s)e^{-\theta_{1}s} & \cdots & g_{1n}(s)e^{-\theta_{n}s} \\ \vdots & \ddots & \vdots \\ g_{n1}(s)e^{-\theta_{n}s} & \cdots & g_{nn}(s)e^{-\theta_{nn}s} \end{bmatrix}$$
(1)  
$$D(s) = \begin{bmatrix} d_{11}(s) & \cdots & d_{1n}(s) \\ \vdots & \ddots & \vdots \\ d_{n1}(s) & \cdots & d_{nn}(s) \end{bmatrix}$$
(2)

The ideal decoupler in Fig. 1 satisfies the following relations.

$$G(s)D(s) = H(s) \tag{3}$$

$$H(s) = \begin{bmatrix} h_{11}(s) & & \\ & \ddots & \\ & & h_m(s) \end{bmatrix}$$
(4)

G(s)adj[G(s)]K(s) = det[G(s)]K(s)Since G, the diagonal D(s) can be obtained by: D(s) can be obtained by:

$$D(s) = adj[G(s)]K(s) = \begin{bmatrix} G^{11}(s) & \cdots & G^{1n}(s) \\ \vdots & \ddots & \vdots \\ G^{n1}(s) & \cdots & G^{nn}(s) \end{bmatrix} K(s)$$
(5)  
$$H(s) = \det[G(s)]K(s)$$
(6)  
$$K(s) = \begin{bmatrix} k_{11}(s) & & \\ & \ddots & \\ & & & k_{nn}(s) \end{bmatrix}$$
(7)

The reliability requirement for the decoupler is that all of its elements must be proper, casual and stable. Form the constructive form of the decoupler, it can be seen that the elements of the decoupler are indeed casual and stable. To make the decoupler realizable, only the properness problem of the elements should be considered. Therefore, a simple stable pole with adequate multiplicity can be inserted as follows.

$$\frac{1}{(\alpha s+1)^m}$$
(8)

where  $\alpha > 0$  and *m* is the minimum positive number that can make all the elements of the

decoupler proper. If  $v_{ii}(s), i = 1, 2, \dots, n$  are chosen as  $v_{ii}(s) = \frac{K_{ii}(s)}{(\alpha s + 1)^m}$ , all the decoupled SISO system. Hence, one can readily obtain that

$$D(s) = \frac{k_{ii}(s)}{(\alpha s + 1)^m} adj[G(s)]$$
<sup>(9)</sup>

$$h_{11}(s) = h_{22}(s) = \dots = h_{nn}(s) = \frac{k(s)\det[G(s)]}{(\alpha s + 1)^m} \triangleq h(s)$$
 (10)

From Eq. (9) and (10), it can be seen that all the decoupled SISO systems are the same. Hence, the controller  $c_{ii}(s)$  are the same. Only one controller needed to be designed.

#### 2.2 H2 Optimal PID Control for SISO Process

Consider the IMC structure in Fig. 2, where g(s) is a SISO process, gm(s) is the model, and q(s) is the IMC controller. A linear plant with output time delay can be described by the transfer function

$$g(s) = \frac{KN_{-}(s)N_{+}(s)}{M_{-}(s)M_{+}(s)}e^{-\theta s}$$
(11)

where K is a real constant that denotes the static gain and  $\theta$  is a positive real constant denoting the pure time delay. The subscript minus sign (-) denotes the roots in the left half-plane and the subscript plus sign (+) denotes the roots in the closed right half-plane; that is, N-(s) and M-(s) are polynomials with roots in the left half-plane, and N+(s) and M+(s) polynomials with roots in the closed right half-plane. It is assumed that

N-(0)=N+(0)=M-(0)=M+(0)=1, and deg{N-(s)}+deg{N+(s)}  $\Box$  deg{M-(s)}+deg{M+(s)}.



Fig. 2. Schematic of the IMC control structure



Fig. 3. Unity feedback control loop

The IMC structure can be related to the unity feedback loop (see Fig. 3) through

$$c(s) = \frac{q(s)}{1 - g(s)q(s)} \tag{12}$$

In the case that the model is exact, that is, g(s)=gm(s), the sensitivity transfer matrix (i.e., the transfer matrix from set point r(s) to the error e(s)) is given by

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$$S(s) = I - g(s)q(s) \tag{13}$$

and the complementary transfer matrix (i.e., the transfer matrix from set point r(s) to the system output y(s)) is given by

$$T(s) = g(s)q(s) \tag{14}$$

Let W1(s) and W2(s) be the performance weighting functions. The performance criteria is chosen as

$$\min \|W_2(s)S(s)W_1(s)\|_2 \tag{15}$$

The idea of the H2 optimal control is to find a controller that stabilizes the system and minimizes the performance index defined in (15). To design the controller, the two weighting functions must be determined first. Instead of choosing complex weighting functions, for step inputs or inputs similar to steps, Zhang et al. gave a reasonable but simple way to determine the weighting functions [15]. The

weighting functions can be chosen as  $W_1(s) = s^{-1}$ , and  $W_2(s) = 1$  Therefore, the performance index of the H2 optimal control can be rewritten as

$$\min \left\| S(s) W_1(s) \right\|_2 \tag{16}$$

The unique H2 optimal IMC controller for the SISO plant with time delay can be obtained by [15]

$$q_{opt}(s) = \frac{M_{-}(s)M_{+}(s)}{KN_{-}(s)N_{+}(-s)}$$
(17)

Since the controller is generally improper, a filter J(s) should be introduced to make the controller proper. For stable plants, J(s) can be chosen as

$$J(s) = \frac{1}{\left(\lambda s + 1\right)^l} \tag{18}$$

where  $\lambda$  is a positive real number and *l* is the relative degree of  $q_{opt}(s)$ . Consequently, the H2 optimal controller can be derived as

$$q(s) = q_{opt}(s)J(s) \tag{19}$$

$$c(s) = \frac{q(s)}{1 - g(s)q(s)} \tag{20}$$

It is easy to verify that c(s) has a pole at the origin. Write c(s) in the following form

$$c(s) = \frac{f(s)}{s} \tag{21}$$

Expand c(s) in a Maclaurin series give

$$c(s) = \frac{1}{s} [f(0) + f'(0)s + \frac{f''(0)}{2!}s^2 + \cdots]$$
(22)

Take the first three terms to approximate the analytical controller. The three terms construct a PID controller as follows.

$$c(s) = K_C (1 + \frac{1}{T_I s} + T_D s)$$
(23)

Here the parameters are given as

$$K_{c} = f'(0), T_{I} = \frac{f'(0)}{f(0)}, T_{D} = \frac{f''(0)}{2f'(0)}$$
(24)

Finally, the H2 optimal PID controller for the decoupled SISO system can be designed. From the design procedure, it can be seen that the method does not need to choose any weighting function. The controller is derived theoretically and given in an analytical form. The parameters of the designed PID controller can be determined easily.

#### **3. TDOF CONTROL STRUCTURE**

In this project, the TDOF control structure shown in Fig. 4 is developed to isolate the disturbance from the reference. Here D(s) is the decoupler, C(s) is the controller of the disturbance loop and C1(s) is the controller of the reference loop.



Fig. 4. Schematic of the TDOF structure

It can be verified that

$$y(s) = G_{R}(s)r(s) + G_{d}(s)d(s)$$
(25)  

$$G_{R}(s) = [I + G(s)D(s)C(s)]^{-1}G(s)D(s)C(s)C_{1}(s)$$
(26)  

$$G_{d}(s) = [I + G(s)D(s)C(s)]^{-1}G_{L}(s)$$
(27)

From Eq. (25)-(27), it can be seen that the effective from d(s) to y(s) can be adjusted only by D(s) and C(s). Therefore, the controller of the disturbance loop should be designed first. After D(s) and C(s) have been designed, the controller of the reference loop C1(s) can be designed as the IMC controller of the obtained closed-loop transfer function matrix.

Since all the decoupled SISO system in this project are the same, it can be derived that T(s) is diagonal and all the elements of T(s) are the same.

$$t_{ii}(s) = \frac{h(s)c(s)}{1 + h(s)c(s)}$$
(28)

Substituting Eq. (10) and Eq. (20) into Eq. (28), one can obtain that

$$t_{ii}(s) = \frac{h(s)c(s)}{1 + h(s)c(s)}$$
(28)

Substituting Eq. (10) and Eq. (20) into Eq. (28), one can obtain that

$$t_{ii}(s) = \frac{N_{+}(s)}{N_{+}(-s)} \frac{1}{(\lambda s+1)^{l}} e^{-\theta s}$$
(29)

Regard  $t_{ii}(s)$  as a new plant and C1(s) as the H2 optimal controller in the next step design. The controller of the reference loop can be obtained as follows:

$$c_{1opt}(s) = (\lambda s + 1)^t \tag{30}$$

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Introduced a diagonal filter J1(s) to the optimal controller:

$$c_1(s) = \frac{(\lambda s + 1)^l}{(\gamma s + 1)^l} \tag{31}$$

Consequently, C1(s) can be derived as

$$C_{1}(s) = \begin{bmatrix} \frac{(\lambda s+1)^{l}}{(\gamma s+1)^{l}} & & \\ & \ddots & \\ & & \frac{(\lambda s+1)^{l}}{(\gamma s+1)^{l}} \end{bmatrix}$$
(32)

#### 4. SIMULATION RESULTS



Fig. 5. Set point tracking and load rejection responses for Tyreus process without model mismatch



Fig. 6. Setpoint tracking and load rejection responses for Tyreus process with model mismatch

#### **5.** CONCLUSIONS

In this project, a novel same controller in all loops control design method based on the ideal decoupler for multivariable stable systems with multiple time delays is proposed. Since the decoupled SISO systems are the same, only one PID controller needed to be designed for the decoupled SISO system. The proposed design method can simplify the design task significantly since all the decoupled loops are the same. Since the disturbance is isolated from the reference, the disturbance rejection can be adjusted only by the parameter  $\Box$  in the first controller and the set-point tracking can be tuned by the parameter  $\Box$  in the second controller after the parameter  $\Box$  is determined. One can trade off between the performance and robustness easily in the proposed control scheme.

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