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An Approach of Membership Function in Fuzzy Transportation Problem

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Abstract: *The object of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. A fuzzy transportation problem basically deals with the problem, which aims to find the best way to fulfil the demand of n demand points using the capacities of m supply points.*

Here we are using the concept of membership function for solving fuzzy transportation problems with mixed constraints and find an optimal solution. The optimal solution procedure is illustrated with numerical example.

1. Introduction

Lotfi Zadeh (1965)[8] introduced the notion of fuzzy sets one of the main difficulties has been with the meaning and measurement of membership functions. Particularly, lack of a consensus on the meaning of membership functions has created some confusion is neither bizarre nor unsound. Over the Years Fuzzy Set Theory has been playing an important role to deal with the problems having uncertainty, vagueness, doubtful data and so on, which cannot be solved with the help of available classical methods. In 1978, Dubois and Prade[3] defined the fuzzy numbers as a fuzzy subset of the real line[1-7]

The commonly accepted theory of fuzzy numbers set up by Dubois and Prade[3] (1978), who proposed a restricted class of membership functions. However, approximation of fuzzy functions and operations are needed if one wants to follow Zadeh's extension principle (1975, 1983). It leads to some drawbacks that concern properties of fuzzy algebraic operations, as well as to unexpected and uncontrollable results of repeatedly applied operations.

2. Membership Function

A fuzzy number is defined by the membership function μ which is defined on the set of real number R. In general, a generalised fuzzy number A described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following condition.

- μ_A is a continuous mapping from R to the closed interval [0,1],
- $\mu_A = 0, -\infty < X \le c.$
- $\mu_A(x) = L(X)$ is strictly increasing on [c,a],
- $\mu_A(x) = W, a \le X \le b,$
- $\mu_A(x) = R(X)$ is strictly decreasing on [b,d],
- $\mu_A = 0, d \le X < \infty,$

Where $0 < W \le 1$, a,b,c and d are real numbers .

3. Membership Function of Trapezoidal Fuzzy Number

A fuzzy number $\widetilde{A}$ is a trapezoidal fuzzy number denoted by $\widetilde{A} = (a_1, a_2, a_3, a_4)$. where a_1, a_2, a_3, a_4 are real numbers and its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } a_2 \le x \le a_3 \\ \dfrac{a_3 - x}{a_4 - a_3} & \text{for } a_3 \le x \le a_4 \\ 0 & \text{for } x > a_4 \end{cases}$$

An alternative concise expression using min and max is:

Trapezoidal (x; a,b,c,d) = $\mathrm{Max}\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$

4. Membership Function of Triangular Fuzzy Number

A number $\tilde{A}$ is a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$. where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } a < x_1 \\ \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

by using min and max, we have an alternative expression for the proceeding equation: triangle (x; a,b,c) = max $\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$

5. Some Examples on Fuzzy Transportation Problem with Membership Function

1. In this problem we have taken the data of the plants of company Hindustan petroleum gas of vidisha district (M.P.). There are three plants (Bhopal, Jubalpur and Indore) of company in M.P. and the supply done by company to three agencies (Atul, Neera and Vidisha) in vidisha. But the problem at the different different time the demand of agencies are different different and as per situation the supply by company can be different. As per situation the new problem arise and known as fuzzy transportation problem. And the problem is:

Table1.

	FD1	FD2	FD3	SUPPLY
FS1	[2,3,4]	[7,8,9]	[1,5,9]	[6,9,12]
FS2	[10,20,30]	[8,16,24]	[3,7,11]	[3,6,9]
FS3	[3,6,9]	[3,4,5]	[3,9,15]	[22,24,26]
FS4	[11,12,13]	[6,16,26]	[4,8,12]	[6,7,8]
DEMAND	[10,15,20]	[15,17,19]	[12,14,16]	[37,46,55]

After applying membership function the problem is:

Table2.

	D1	D2	D3	SUPPLY
S1	[α+2,3-α]	[α+7,9-α]	[4α+1,9-4α]	[3α+6,12-3α]
S2	[10α+10,30-10α]	[8α+8,24-8α]	[4α+3,11-4α]	[3α+3,9-3α]
S3	[3α+3,9-3α]	[α+3,5-α]	[6α+3,15-6α]	[2α+22,26-2α]
S4	[α+11,13-α]	[10α+6,26-10α]	[4α+4,12-4α]	[α+6,8-α]
DEMAND	[5α+10,20-5α]	[2α+15,19-2α]	[2α+12,16-2α]	TOTAL [9α+37,55-9α]

Now we are using separation method in above problem and considering lower bound fuzzy transportation problem in integer form

Table3

	FD1	FD2	FD3	SUPPLY
FS1	[α+2]	[α+7]	[4α+1]	[3α+6]
FS2	[10α+10]	[8α+8]	[4α+3]	[3α+3]
FS3	[3α+3]	[α+3]	[6α+3]	[2α+22]
FS4	[α+11]	[10α+6]	[4α+4]	[α+6]
DEMAND	[5α+10]	[2α+15]	[2α+12]	TOTAL [9α+37]

After applying vogel approximation method the problem is:

Table4.

	FD1	FD2	FD3	SUPPLY
FS1	**[5α+3]** [α+2]	[α+7]	**[-2α+3]** [4α+1]	[3α+6]
FS2	[10α+10]	[8α+8]	**[3α+3]** [4α+3]	[3α+3]
FS3	**7** [3α+3]	**[2α+15]** [α+3]	[6α+3]	[2α+22]
FS4	[α+11]	[10α+6]	**[α+6]** [4α+4]	[α+6]
DEMAND	[5α+10]	[2α+15]	[2α+12]	TOTAL [9α+37]

Now the fuzzy transportation cost (in integer form) is

$= (5\alpha+3)(\alpha+2) + (-2\alpha+3)(4\alpha+1) + (3\alpha+3)(4\alpha+3) + (7)(3\alpha+3) + (2\alpha+15)(\alpha+3) + (\alpha+6)(4\alpha+4)$

$= [5\alpha^2 + 13\alpha + 6] + [-8\alpha^2 +10\alpha +3] + [12\alpha^2+21\alpha + 9] + [21\alpha + 21] + [2\alpha^2 + 21\alpha + 45] + [4\alpha^2 + 28\alpha + 24]$

$= [15\alpha^2 + 114\alpha + 108]$

And the upper bound fuzzy transportation problem in integer form is

Table5.

	D1	D2	D3	SUPPLY
S1	[3-α]	[9-α]	[9-4α]	[12-3α]
S2	[30-10α]	[24-8α]	[11-4α]	[9-3α]
S3	[9-3α]	[5-α]	[15-6α]	[26-2α]
S4	[13-α]	[26-10α]	[12-4α]	[8-α]
DEMAND	[20-5α]	[19-2α]	[16-2α]	TOTAL [55-9α]

In same above manner after applying vogel approximation the fuzzy transportation problem is

Table6.

	D1	D2	D3	SUPPLY
S1	**[12-3α]** [3-α]	[9-α]	[9-4α]	[12-3α]
S2	[30-10α]	[24-8α]	**[9-3α]** [11-4α]	[9-3α]
S3	**[7]** [9-3α]	**[19-2α]** [5-α]	[15-6α]	[26-2α]
S4	**[1-2α]** [13-α]	[26-10α]	**[7+α]** [12-4α]	[8-α]
DEMAND	[20-5α]	[19-2α]	[16-2α]	TOTAL [55-9α]

Transportation cost of fuzzy transportation problem in integer form is

$[12 – 3\alpha][3-\alpha] + [9-3\alpha][11-4\alpha] + [7][9-3\alpha] + [19-2\alpha][5-\alpha] + [1-2\alpha][13-\alpha] + [7+\alpha][12-4\alpha]$

$[36 - 21\alpha + 3\alpha^2] + [99 - 69\alpha + 12\alpha^2] + [63 - 21\alpha] + [95 - 29\alpha + 2\alpha^2] +$

$[13-27\alpha +2\alpha^2] + [84 - 16\alpha -4\alpha^2]$

$= [15\alpha^2 -177\alpha + 390]$

Thus the fuzzy optimal solution for the problem is

$[15\alpha^2 + 114\alpha + 108, 15\alpha^2 -177\alpha + 390]$

Special cases:

- If $\alpha = 0$

Then T.C. in interval form is = [108,390]

- If $\alpha = 0.5$

Then T.C. in interval form is = [168.75, 305.25]

- If $\alpha = 0.75$

Then T.C. in interval form is = [201.9375, 265.6875]

- If $\alpha = 1$

Then T.C. in interval form is = [237,228]

Problem 2: A company has four sources S_1, S_2, S_3 and S_4 and four destinations D_1, D_2, D_3 and D_4; the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is C_{ij} where

$$[C_{ij}]_{3\times 4} = \begin{pmatrix} (1,2,3,4),(1,3,5,6),(9,11,12,14),(5,7,8,11) \\ (0,1,2,4),(-1,0,1,2),(5,6,7,8),(0,1,2,3) \\ (3,5,6,8),(5,8,9,12),(12,15,16,19),(7,9,10,12) \end{pmatrix}$$

The fuzzy availability of the product at source are ((1,6,7,12), (0,1,2,3), (5,10,12,17),) and the fuzzy demand of the product at destinations are ((5,7,8,10), (1,5,6,10), (1,3,4,6) (1,2,3,4)) respectively.

Then the problem become as:

Table7.

	D1	D2	D3	D4	SUPPLY
S1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
S2	(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
S3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
DEMAND	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

After applying the formula of membership function the problem is:

Table 8.

	D1	D2	D3	SUPPLY
S1	5α+5, 20-5α	3α+3, 12-3α	α + 1, 4-α	5α+2, 35-5α
S2	2α+2, 8-2α	10α+10,40-10α	α+ 2 , 5-α	α + 17, 20-α
S3	4α+4, 16-4α	α + 17, 20-α	2α+6,12-2α	2α+10,16-2α
S4	7α+7, 28-7α	α + 8, 11-α	2α+9, 15-2α	α +8 , 11- α
DEMAND	3α+3,28-7α	5α+30, 45-5α	α + 22, 25-α	T= 9α + 55, 82-9α

After using separation method in above fuzzy transportation problem and considering lower bound fuzzy transportation problem in integer form the problem is

Table 9.

	D1	D2	D3	SUPPLY
S1	5α+5	3α+3	α +1	5α +20
S2	2α +2	10α +10	α +2	α +17
S3	4α +4	α +17	2α + 6	2α +10
S4	7α + 7	α + 8	2α + 9	α + 8
DEMAND	3α + 3	5α + 30	α + 22	T= 9α + 55

After Applying VAM

Table10.

	D1	D2	D3	SUPPLY
S1	5α+5	**5α +20** 3α+3	α +1	5α +20
S2	2α +2	10α +10	**α +17** α +2	α +17
S2	2α +2	10α +10	**α +17** α +2	α +17
S3	**3α+3** 4α +4	**2-α** α +17	**5** 2α + 6	2α +10
S4	7α + 7	**α + 8** α + 8	2α + 9	α + 8
DEMAND	3α + 3	5α + 30	α + 22	T= 9α + 55

The transportation cost is:

$(5\alpha + 20)(3\alpha+3) + (\alpha + 17)(\alpha + 2) + (\alpha + 8)(\alpha + 8) + (3\alpha+3)(4\alpha+4) + (2-\alpha)(\alpha+17) + (5)(2\alpha+6)$

$= [28\alpha^2 + 140\alpha + 234]$

And the lower bound fuzzy transportation problem in integer form is

Table11.

	D1	D2	D3	SUPPLY
S1	[20-5α]	[12-3α]	[4-α]	[35-5α]
S2	[8-2α]	[40-10α]	[5-α]	[20-α]
S3	[16-4α]	[20-α]	[12-2α]	[16-2α]
S4	[28-7α]	[11-α]	[15-2α]	[11-α]
DEMAND	[12-3α]	[45-5α]	[25-α]	TOTAL [82-9α]

In same above manner after applying vogel approximation the fuzzy transportation problem is

Table12.

	D1	D2	D3	SUPPLY
S1	[20-5α]	[10-4α] [12-3α]	[25-α] [4-α]	[35-5α]
S2	[12-3α] [8-2α]	[8+2α] [40-10α]	[5-α]	[20-α]
S3	[16-4α]	[16-2α] [20-α]	[12-2α]	[16-2α]
S4	[28-7α]	[11-α] [11-α]	[15-2α]	[11-α]
Demand	[12-3α]	[45-5α]	[25-α]	TOTAL [82-9α]

Transportation cost of problem in integer form is:

$[10 - 4\alpha][12-3\alpha] + [25-\alpha][4-\alpha] + [12-3\alpha][8-2\alpha] + [8+2\alpha][40-10\alpha] + [16-2\alpha][20-\alpha] + [11-\alpha][11-\alpha]$

$[120 - 78\alpha + 12\alpha^2] + [100 - 209\alpha + \alpha^2] + [320 - 20\alpha] + [96 - 48\alpha + 6\alpha^2] +$

$[320-40\alpha + 2\alpha^2] + [121 - 22\alpha + \alpha^2]$

$= [2\alpha^2 - 217\alpha + 1077]$

Thus the fuzzy optimal solution for the fuzzy transportation problem is:

$[28\alpha^2 + 140\alpha + 234,\ 2\alpha^2 - 217\alpha + 1077]$

Special cases:

- If $\alpha = 0$
  Then T.C. in interval form is = [234, 1077]
- If $\alpha = 0.5$
  Then T.C. in interval form is = [311,969]
- If $\alpha = 0.75$
  Then T.C. in interval form is = [354.75, 915.375]
- If $\alpha = 1$
  Then T.C. in interval form is = [402,862]

6. CONCLUSION

The membership function is very easy and comfort for solving any fuzzy transportation problem but drawback of this method is that we can't solve every fuzzy transportation problem with this method. I.e. We can solve only that problems in which $a_1=b_1$, $a_2=b_2$, $a_3=b_3$, $a_4=b_4$. If problem is balanced but numbers are not equal like $a_1\neq b_1, a_2\neq b_2$, $a_3\neq b_3$, $a_4\neq b_4$ or $a_1=b_1, a_2\neq b_2$, $a_3\neq b_3$, $a_4=b_4$ or $a_1\neq b_1, a_2=b_2$, $a_3=b_3$, $a_4\neq b_4$ then we can't solve this type of problem with membership function.

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